European University Institute

DEPARTMENT OF ECONOMICS

Topics in Quantitative Macroeconomics

Dominik Menno

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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Für Cornelia und Isolde.

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Abstract

This thesis contributes to two traditional debates in quantitative macroeconomics: (i) the welfare costs of aggregate shocks and (ii) the role of financial integration for international business cycle co-movement and risk-sharing. In recent years both debates regained attention, in particular after the experience of the recent U.S. financial crisis and the subsequent international recession.

The first chapter of this thesis, joint work with Tommaso Oliviero, investigates the welfare effects of the U.S. Great Recession. Motivated by evidence that more leveraged households lost more in terms of housing wealth during the recent recession, we quantify the welfare effects of the Great Recession for two types of households, namely borrowers and savers. We simulate the Great Recession as a contemporaneous negative shock to aggregate income and the efficiency of the financial intermediation sector. The latter moves the interest rates for debt and is therefore the main driver behind households' leverage. We find that in the Great recession borrowers lose significantly more in terms of welfare than savers. In counter-factual experiments we find this loss to be larger the higher the households' leverage. This last effect comes from non-linearity that is absent in a model with an always binding collateral constraint (i.e. constant leverage).

The second chapter contributes to the debate on the role of financial integration for international business cycles. For the G7 countries, I document that country pairs with more bilateral FDI linkages have more synchronized investment cycles. I also find that the relation between FDI integration and synchronization of gross domestic product (GDP) is - yet positive - statistically insignificant after controlling for time fixed effects. I then study a model of international business cycles with an essential role for FDI and shocks to multinational activity. In the model, more FDI openness unambiguously increases investment synchronization while the effect on GDP synchronization is ambivalent. Due to mismeasurement of intangible capital in national accounts, the actual elasticity of output synchronization with respect to FDI integration is underestimated. The effects measured in the data are quantitatively consistent with the model predictions. The model also has important implications for consumption risk sharing. Finally, shocks to multinational activity have the potential to resolve the so called 'quantity puzzle' in international macroeconomics.

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Overview of the thesis chapters

This thesis contributes to the literature by investigating the welfare consequences of aggregate shocks and the role of financial integration for international business cycles. Both topics have a long tradition in macroeconomic research and have regained interest with the events starting in 2007 that led to the U.S. Great Recession and eventually turned into an international recession.

The first chapter of this thesis, joint work with Tommaso Oliviero, investigates the welfare consequences of the U.S. Great Recession. Using Panel data evidence from the Survey of Consumer Finances 2007/09, we document that households that were more leveraged in 2007 experienced a bigger loss in housing wealth during the recession. We then quantify the welfare effects of the observed drop in house prices during the Great Recession for leveraged and un-leveraged households. For this purpose, calibrate a dynamic general equilibrium model with heterogeneous households to the U.S. economy and simulate the Great Recession as a contemporaneous negative shock to aggregate income and a negative shock to the financial intermediation sector. Intermediation shocks affect the ability of the banking sector to transform deposits into credit and therefore drive households' leverage. Aggregate income shocks account, instead, for the observed drop in house prices. There are three main lessons from the theory. First, in the Great recession leveraged households (borrowers) experience significantly bigger welfare losses than un-leveraged agents (savers). Second, intermediation shocks have, by their nature, re-distributive effects by transferring wealth from borrowers to savers. Third, leverage plays an important role for welfare: we estimate that a significant part of borrowers' welfare loss comes from the fact the leverage was high when the recession hit the economy. This last effect is absent in a model with an always binding collateral constraint (as often assumed in the literature).

The second chapter studies the effect of foreign direct investment (FDI) through multinationals on the international transmission of shocks. For the G7 country pairs, I find

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that increases in bilateral FDI linkages are associated with more synchronized investment cycles. This is an important finding because one of the most robust predictions of the standard international business cycle model is the strong negative correlation of investment when financial markets integrate. With respect to aggregate activity, I find that the relation between FDI integration and synchronization of gross domestic product (GDP) is - yet positive - statistically insignificant, confirming the literature that is inconclusive on this issue. I then study a model of international business cycles with an essential role for FDI and shocks to multinational activity. Multinationals accumulate technology capital that can be contemporaneously used in all plants the multinational operates, independent whether located in the home country or abroad. In addition, the productivity of multinationals is stochastic and gets transferred to all plants the multinational operates. Therefore, multinationals itself act as a source of business cycles volatility - on top of affecting the propagation mechanism of country-specific shocks that originate in other sectors of the economy. The model predicts that more FDI openness unambiguously increases investment synchronization. It also offers a rationale for the weak link between FDI integration and GDP synchronization: In national accounts, GDP is mismeasured because intangible capital is expensed. Due to this mismeasurement, the actual elasticity of output synchronization with respect to FDI integration is underestimated. Finally, I use the model to assess the risk sharing implications when countries open up for FDI. I show that even when financial markets are complete, FDI integration reduces the consumption risk to which households are exposed by mitigating the impact of country-specific shocks on production. This reduction in aggregate risk is strongest in the model with country-specific shocks only. Shocks to multinational activity, on the other side, counter-act the risk-reducing effect of more FDI linkages because production in both countries is increasingly determined by shocks to multinationals. To the best of my knowledge, I am the first one to make this point in a fully dynamic business cycle framework. This last point is also related to the so called 'quantity puzzle' in international macroeconomics that refers to the counter-factual ranking of cross-country correlations of GDP and consumption in standard business cycle models. When households cannot trade any financial assets, the model with shocks to multinational activity generates the observed ranking in the data because households are left with a large part of risk that is uninsured.

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Chapter 1

Financial Intermediation, House Prices, and the Welfare Effects of the U.S. Great Recession

With Tommaso Oliviero

1.1 Introduction

The U.S. Great Recession was characterized by a large fall in GDP coupled with an unprecedented collapse in the housing market. This drop in aggregate house price between 2007:IV and 2009:II deeply affected a great number of U.S. households. [1] Figure 1.1 shows the de-trended quarterly series of US GDP and aggregate house prices. We observe a large drop of around 5.4% between the NBER recession dates, and a collapse in aggregate house prices of about 11%.

The recession has also been linked turbulence in the financial markets and, in particular, the banking system. This fact has triggered a debate among economists and policy-makers about the welfare consequences of the financial innovation process that preceded the crisis and that possibly exacerbated the effects of the economic collapse. In fact, the last decade witnessed an increase in household indebtedness that coincided with a period of relaxing credit conditions. Both microeconomic and macroecronomic evidence show an increase in household leverage in the years preceding the recession. On the micro side, an analysis of Survey of Consumer Finance (SCF) data reveals that aggregate mortgage debt

[1] (Iacoviello, 2011b) shows that housing wealth represents about half of total household net worth in 2008 and almost two third of median household total wealth

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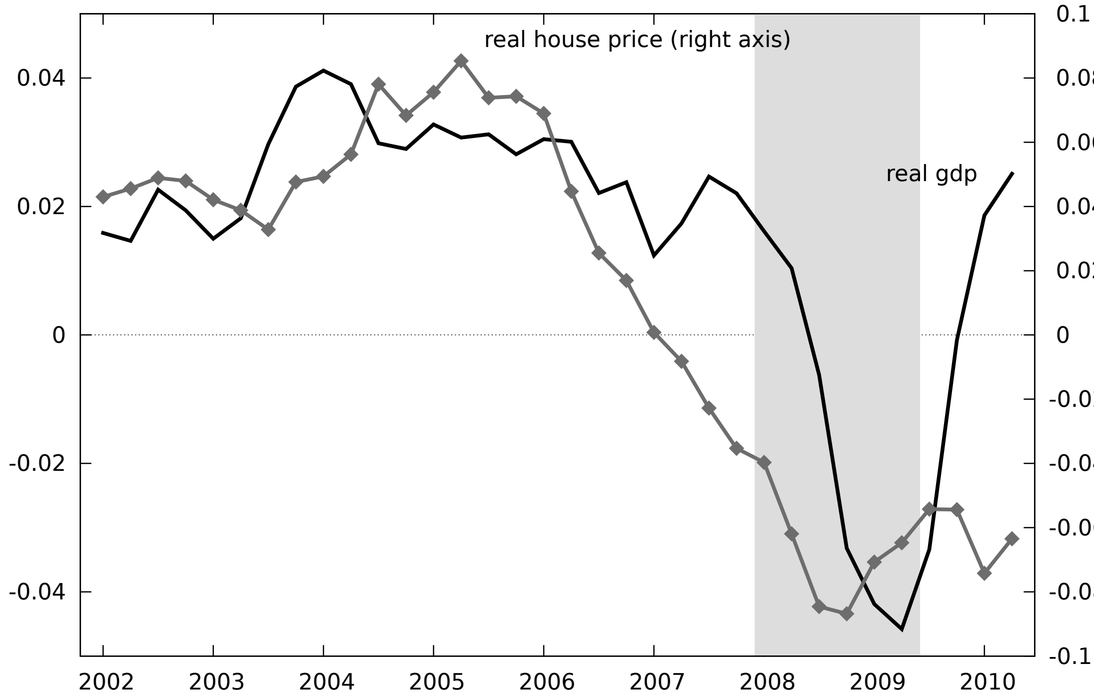
Figure 1.1: GDP growth vs. house price growth



Notes: Shaded areas are NBER recession dates. The grey dotted-line is the Y2Y-growth rate of All-Transactions House Price Index for the United States deflated by CPI (less shelter); the black line is Y2Y growth of U.S. real GDP. For a detailed data description see appendix A.1.

expanded by 59% between 2001 and 2007, despite a 19% increase in housing wealth. On the macro side, we observe around ten quarters of growth in leverage followed by sharp fall during the NBER recession dates, as seen in the mortgage to real estate ratio. Figure 1.2 plots the year-to-year growth rate of leverage and the spread between the mortgage interest rate and the federal funds rate. These two series show a negative correlation at the onset, and in the last quarters of the Great Recession. During the quarters preceding the crisis, spreads were particularly low and leverage was rising at an unprecedented rate. In mid-2008 however, interest rate spreads jumped to a level of about 4.5% while household leverage started to decline. Our interpretation is that, in the period of credit expansion (low spreads), the mortgage growth rate was faster than real estate inflation and leverage was increasing; the opposite happened in a period of credit contraction (high spreads).

In the current paper we examine the effects of exogenous changes in interest rate spreads on endogenous aggregate house prices and, ultimately, on households' welfare. In this respect, we share the view that fluctuations in spreads largely reflect disturbances in the financial markets' assessments of credit risk (Bordo, 2008). Furthermore, we share the view of Adrian and Shin (2010) that variations in the price of default risk reflected

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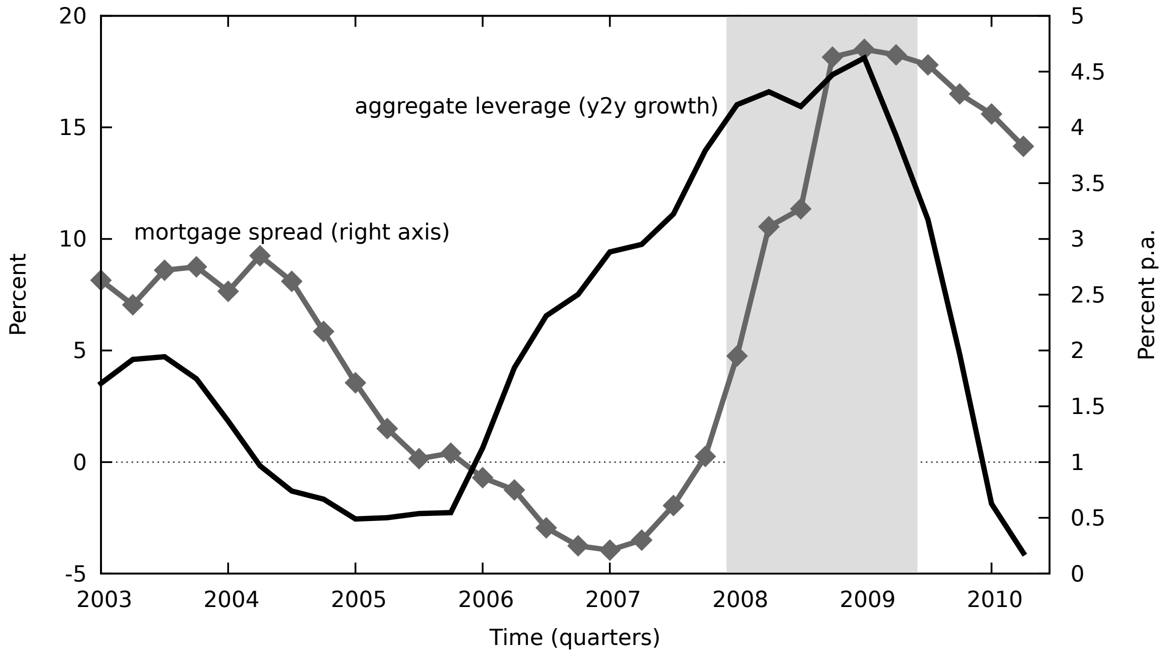
Figure 1.2: Mortgage spread vs. Leverage



Notes: Shaded areas are NBER recession dates. The grey dotted-line shows the spread between the one-year amortizing adjustable mortgage rate (ARM) and the federal funds rate from 2002:I to 2010:II. Spreads of ARM over Fed Funds rate are shown in levels (percent p.a.). The black line is Y2Y growth of U.S. leverage defined as the ratio between mortgage and real estate series (taken from the balance sheet of U.S. households and nonprofit organizations). For a detailed data description see appendix A.1.

variations in the effective risk-bearing capacity of the financial sector, which has been ultimately affected by aggregate portfolio losses.

The stylized facts highlighted in figures 1.1 and 1.2 motivate our interest in quantifying and isolating the impacts of financial and income shocks on aggregate house prices and, consequently, on households' welfare. In particular we address this question within a stochastic dynamic general equilibrium model with heterogeneous households and endogenous collateral constraints. In our model, households differ in their level of patience. This heterogeneity results into two types of agents: borrowers, who are potentially financial constrained; and savers, who are unconstrained.[2] Within this framework, we study the welfare effects of an endogenous drop in housing wealth for these two groups of households. The data in table 1.1 motivate the choice of this cross-sectional heterogeneity across households. Using panel data from the Survey of Consumer Finance (SCF) for the period from 2007 to 2009, the table shows that households with a positive net savings position (*savers*) show an average drop in housing wealth of 9.2% between 2007 and 2009.

[2]The structure of the economy is similar to Iacoviello (2011a) and Justiniano, Primiceri, and Tambalotti (2013) who present a quantitative analysis of the US Great Recession

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This is significantly lower than the equivalent number for households with a negative net savings position (*borrowers*, -16.6%).[3] Moreover we show that the drop in housing wealth for borrowers is increasing in the level of leverage in 2007:[4] while borrowers with initial levels of leverage greater than or equal to 67% show a drop of 23.5% in housing wealth, households that entered the recession with a lower level of leverage (less than 43%) show a much smaller drop in housing wealth.

Table 1.1: Summary Statistics from SCF panel 2007-2009

Household type$_{2007}$	*Savers*	*Borrowers*	*All households*
$\Delta_{07,09}$housing wealth	-9.2%	-16.6%	-12.9%
Leverage$_{2007}$	*< 43%*	*43 - 67%*	*>67%*
$\Delta_{07,09}$housing wealth	-12.9%	-16.5%	-23.5 %

In the model economy, agents are fully rational and derive utility from both the consumption of perishable goods and of housing services coming from housing stock. Housing is the only physical asset in the economy and it is fixed in supply. This is motivated by the fact that previous and during the Great Recession, house prices were most volatile in geographical areas where the supply of houses was relatively fixed.[5] The financial friction arises because agents have to collateralize short positions of one-period financial asset by a fraction of the expected value of their available housing stock.

In this otherwise standard model, we introduce a competitive financial intermediation sector. All saving and borrowing is conducted though this sector, which faces exogenous shocks to its technology.[6] These shocks give rise to a spread between borrowing and

[3] In table 1.1, saver and borrower status refers to households in 2007. Savers and borrowers are defined here - and throughout the paper- as households that show respectively a positive or a negative net asset position. A net asset position is defined as the sum of savings bonds, directly held bonds, the cash value of life insurances, certificates of deposits, quasi-liquid retirement accounts and all other types of transaction accounts minus the debt secured by primary residence, the debt secured by other residential property, credit card debt and other forms of debt. For a detailed description of data please refer to Appendix A.1

[4] Leverage is defined here - and thorough the remaining sections- as the ratio between net asset position and total housing wealth.

[5] See figure IV in Mian and Sufi (2009).

[6] We consider a simple model for the financial intermediation in the spirit of Cooper and Ejarque (2000) and Cúrdia and Woodford (2010). Otherwise, the link to these studies is limited as the former looks at the business cycle properties of financial shocks within a representative agent framework, while the latter studies the implications of spread shocks for the optimal conduct of monetary policy.

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lending such that the collateral constraint does not necessarily bind. In other words, it generates endogenous changes in the households' leverage. The second source of aggregate disturbance comes from standard aggregate income shocks that directly affect the households' endowment of the perishable good. This may be interpreted as a reduced form way to capture the cyclical behavior of productivity shocks.

We calibrate the model to the US economy and simulate the Great Recession as a contemporaneous negative income and financial shock that follows a period of moderate economic, credit expansion and increasing leverage. This characterization is due to the empirical observation that both income and financial intermediation were above (below) the long run trend before (after) the recession. To calibrate our key parameters we consider moments from both micro and macro data. In particular, we were able to match the leverage and the wealth share of borrowers relative to savers using from the Survey of Consumer Finances (SCF, waves 1998 - 2007). This calibration strategy, although different from the approach of most papers in the existing literature which target macro moments only, results in calibrated parameters that are compatible with recent contributions (Iacoviello and Guerrieri (2012)).

A very delicate issue for the calibration exercise is what time frame to use, and in particular, whether to incorporate a recession or not. We take the following stance. Our main goal is to maintain a close link between the model and the research question. We study the Great Recession as a state-contingent exogenous event that hit the US economy in late 2007, following a period characterized by banking innovation and increasing household leverage. Therefore, we consider the Great Recession as a low probabilistic event embedded in a business cycle framework. For this reason, we calibrate the model to data including the quarters of the recession until 2009:II.[7] The structural nature of our exercise allows us to conduct counter-factual experiments in order to disentangle the quantitative effects of income and intermediation shocks on aggregate house prices and agents' welfare.

We have three major findings. First, we find that our benchmark model quantitatively explains the observed drop in house prices during the Great Recession. The majority of the effect is attributed to real income shocks. Financial intermediation shocks explain only a small percentage of the observed drop. This finding confirms that the observed behavior of aggregate house prices, before and after the Recession, could be partially related to changes in fully expected shocks. More importantly, we find that, in contrast

[7]For the micro data, SCF is run every three years. We decided to include the 2009 wave and not to include the 2010 wave of the survey in the analysis in order to be consistent with the other calibrated parameters in the model. However, even when including the 2010 wave, the targeted values are very similar.

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to the widespread view, shocks in the financial sector have very limited quantitative effects on aggregate house prices.

Second, we find that borrowers significantly lost more than savers in the Great Recession. In particular we highlight a significant difference in the welfare effects of income and financial intermediation shocks. In the Great Recession, the negative income shock was the main driver behind the absolute drop in house prices and the absolute level of agents' welfare losses. The financial intermediation shock is instead the main determinant of changes in households' leverage before and after the house price drop.[8]. We show that increasing interest spreads had distributive effects, with savers gaining at the expense of borrowers. Accordingly we show that an increase in interest rate spreads forced borrowers to de-leverage and amplified their welfare losses of house price drop by 37.5% while causing a 66.7% welfare gain for savers. Moreover, counterfactual experiments show that the high leverage previous to the crisis made borrowers' welfare losses 25% bigger than if it would have occurred in a state of low leverage.

Third, we find that if we restrict the collateral constraint so that it always binds, the amplification effects given by leverage and de-leverage would have been underestimated; a model with always binding collateral constraint which reduces in fact the volatility of the aggregate leverage to zero. This is an important finding as previous studies (notably, Iacoviello (2005)) usually assume that the constraints are always binding. The intuition for this result is that when the growth rate of the borrowers' debt is forced to be proportional to changes in expected housing wealth, borrowers leverage up more slowly in expansions and de-leverage more slowly in contractions when compared to our benchmark model. This implies that when the crisis hits, borrowers have more outstanding debt in the benchmark model that they need to roll-over. In a recent paper, Iacoviello and Guerrieri (2012) explore the quantitative properties of occasionally binding collateral constraints and the relative non-linear effects coming from changes in the demand for housing.

The mechanism behind the three findings is the following. First, a negative realization of one or both of the exogenous shocks leads to credit contractions. In a credit contraction - given that it is more costly to roll over existing debt - borrowers choose optimally to reduce their indebtedness. If the reduction in debt is sufficiently large, borrowers need to reduce their housing stock. For a given supply of housing, house prices must therefore decrease. This causes borrowers to suffer in terms of both wealth and expected lifetime utility. On the other hand - because of the lower demand for debt - savers potentially

[8]This mechanism is in line with the microeconomic evidence of Mian and Sufi (2010), who found that an increase in credit supply, coupled with the effect of collateralized debt on increasing house prices, created an unprecedented increase in household leverage in the quarters preceding the crisis

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face a lower interest rate on savings. This potentially hurts them by raising the price of future consumption. However, savers expecting house prices to rise again in the next period - can smooth their consumption by buying houses when their prices are depressed. Finally, savers gain in terms of wealth and do not suffer much in terms of expected lifetime utility. The size of this distributive effect depends crucially on how interest rates move. In this paper we quantitatively show what exactly distinguishes financial shocks from income shocks. Another important remark concerns the non-linearity generated by the collateral constraint. In states of the world where borrowers choose optimally to move away from the constraint, it becomes slack. That is, borrowers can choose the pace at which to reduce their debt, unlike the case in models with an always-binding constraint. This implies a change in the elasticity of the demand for debt and housing with respect to changes in house prices that could have non-negligible quantitative effects.

The present study is related to two important strands of literature. First, we relate to the recent literature that studies the financial sector as an autonomous source of macroeconomic fluctuations (Quadrini and Jermann, 2012) and the literature that claims that financial frictions played a pre-eminent role in explaining the observed drop in US aggregate economic activity (Hall, 2011). Recently, Guerrieri and Lorenzoni (2011) find that a shock to the spread between the interest rate on borrowings and the interest rate on savings - in the presence of a collateral constraint that links debt to the level of durables - generates a decrease in the borrowers' demand for durables that grows stronger as agents get closer to the credit constraint. While their analysis abstracts from aggregate house prices and endogenous changes in wealth, we explicitly emphasize the channel that goes through the endogenous change in house prices.

Second, our analysis relates to recent studies on the distributive effects of the Great Recession. Compared to Glover, Heathcote, Krueger, and Ríos-Rull (2011) - a study on intergenerational redistribution during the Great Recession - we focus on a different dimension of agent heterogeneity and welfare, namely, redistribution between constrained agents (borrowers) and unconstrained agents (savers). Similar to Hur (2012), we find that the constrained agents always lose more than unconstrained agents.[9] Both of the aforementioned studies are silent about the inherent redistributive nature of financial shocks, the focus of this paper.[10]

The rest of the paper is structured as follows: In the following section we present the

[9] Hur (2012) considers an overlapping generations model with collateral constraints; he finds that the constrained agents are mostly from the young cohort, and that those agents suffer the most during a recession.

[10] Another distinguishing element of our analysis to Hur (2012) and Guerrieri and Lorenzoni (2011), is that they consider the recession as an unanticipated event while, in our economy, agents take into account the probability of negative aggregate shocks when making decisions about the future.

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model. Section 1.3 presents the quantitative analysis. In section 1.4 we compare the predictions of the benchmark model to alternative specifications, including the case of an always binding constraint. Section 1.5 concludes.

1.2 Model

1.2.1 The physical economy

Uncertainty. Time is discrete and denoted by $t = 0, 1, \ldots$. In each period t, the world experiences one of Z possible exogenous events $z \in \mathcal{Z} = \{1, \ldots, Z\}$. The resolution of uncertainty is represented by an event tree Σ with root σ_0, which is given by a fixed event z_0 in which the economy starts at time 0. Each node is characterized by a history of events, denoted by $\sigma^t = (\sigma_0, \ldots, \sigma_t) \in \Sigma^t = \times_{k=0}^{t} \Sigma_k$. Each node has Z immediate successors $(\sigma_t z^+)$ and a unique predecessor (σ_t^-). The exogenous events follow a Markov process with transition matrix Π.

Agents and Endowments At each node σ_t there are two types of agents, borrowers (denoted by a subscript b) and savers (denoted by a subscript s). Borrowers and savers differ in their rates of time preference, in the sense that borrowers discount the future more than savers. Formally, we have $\beta_s > \beta_b$, where $\beta_i \in (0, 1)$ for $i = s, b$. Each group consists of infinitely many agents but the group size differs: denote by n_b and n_s the relative size of the borrower and saver groups. Note that we choose the normalization $n_b + n_s = 1$.

At each node σ_t, there is a perishable consumption good (non-durable consumption good). The total endowment of the perishable good is stochastic and depends on the realization of the shock alone, that is, $y(\sigma_t^-) = y(z)$, where $y : \mathcal{Z} \to R_{++}$ is a time-invariant function. Note that there is no idiosyncratic uncertainty, the endowment of the perishable good is the same for both types of households. In addition to the non-durable consumption good, agents trade houses. Houses are the only physical asset in the economy and are in fixed net supply. This is motivated by the fact that house prices were most volatile in counties where the supply of houses remained relatively fixed as shown by Mian and Sufi (2010). At period 0, agent $i = b, s$ owns a stock $h_i(\sigma_0^-) \geq 0$ of houses. We normalize $\sum_{i=b,s} h_i(\sigma_0^-) = 1$.

At node σ_t let $h_i(\sigma_t)$ denote agent $i's$ end-of-period stock of houses. We assume that houses are traded cum services. That is, buying a house allows the agent to enjoy the housing services in the same period: if agent i owns $h_i(\sigma_t)$ houses then he receives a

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service stream of $1 \cdot h_i(\sigma_t)$. Other than the service stream, houses do not yield any dividend payments.[11]

Markets. At each node, spot markets open and agents trade the perishable consumption good. We choose the perishable good as the numeraire and - without loss of generality - normalize its price to be equal to 1. Agents can trade housing in every period; that is, agents $i = s, b$ can buy a unit of housing at node σ_t at price $q(\sigma_t)$. As long as $h_i \geq 0$, there is no possibility of default since no promises are made when agents hold a positive amount of the physical asset. In addition to houses, there are two financial assets, debt and savings, both one-period securities. We denote agent i's end-of-period debt holdings by $d_i(\sigma_t)$ and end-of-period savings by $s_i(\sigma_t)$, respectively. Denote the prices of the respective securities by $p_j(\sigma_t)$ for $j = d, s$. We distinguish these two assets because their effective returns differ. Debt is assumed to be a security for which only negative (short) positions are allowed, that is, $d_i(\sigma_t) \leq 0$. For savings, agents can only take positive (long) positions, such that $s_i(\sigma_t) \geq 0$, for $i = b, s$ and all σ_t. Asset $j = d, s$ traded at σ_t promises a nominal pay-off $b_j(\sigma_t z)$ at any successor node $\sigma_t z$. We normalize $b_j(\sigma_t z) = 1$ for all $\sigma_t, \sigma_t z$. For the remainder of the paper, we will discuss pay offs in in terms of real interest rates: denote by $R_D(\sigma_t) = \frac{1}{p_d(\sigma_t)}$ the real interest rate on debt and $R(\sigma_t) = \frac{1}{p_s(\sigma_t)}$ the real interest rate on savings. We also restrict borrowers to hold zero savings and savers to hold zero debt. Formally, for all nodes σ_t, we have $d_b(\sigma_t) \leq 0$, $s_b(\sigma_t) = 0$, $d_s(\sigma_t) = 0$, and $s_s(\sigma_t) \geq 0$.[12]

Collateral Requirements and Default. Similar to Kiyotaki and Moore (1997) we assume limits on debt obligations. Houses are distinguished from other assets by the fact that they are widely used as collateral for debt obligations (mortgages). As in Iacoviello and Neri (2010), the theoretical justification for collateral constraints is the ability of borrowers to default on their debt promises. If the borrowers default in some successor node $\sigma_t z^+$, lenders can seize the borrowers' assets, $q(\sigma_t z^+)h_b(\sigma_t)$ by paying a proportional transaction cost of $(1-m)E[q(\sigma_t z^+)|\sigma_t]h_b(\sigma_t)$ that is not redistributed. This transaction cost can be thought of as a loss associated with bankruptcy. Lenders will therefore never accept a debt contract where the borrowers' promises exceed the expected collateral value

[11]These assumptions are for simplicity. We could allow the service stream of houses to depend on the realization of the shock z or on the identity of the agent.

[12]This is only for the ease of exposition. When computing the equilibrium policy functions, we allow borrowers and savers to trade both assets, debt and savings. Borrowers will only want to take long positions in savings for high relative wealth shares. In the calibrated economy, this never occurs along the equilibrium path unless the initial wealth share of the borrowers is very high.

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of housing. Formally, in each node σ_t, promises made by the borrower have to satisfy

$$R_D(\sigma_t)d(\sigma_t) + mE[q(\sigma_t z^+)|\sigma_t]h_b(\sigma_t) \geq 0. \tag{1.1}$$

Note that in some successor node $\tilde{z} \in \sigma_t z^+$ it might still be optimal for the borrowers to default ex-post. We assume throughout the analysis, however, that m is small enough that borrowers will never default in equilibrium:

Assumption 1

$$m \leq \frac{\min\left(q(\sigma_t z^+)\right)}{E[q(\sigma_t z^+)|\sigma_t]} \qquad \textit{for all } \sigma_t.$$

There is no default in equilibrium if and only if this condition is satisfied.[13] When solving the model equilibrium numerically, we assume that this condition holds and verify ex post that it is indeed satisfied for all prices along the equilibrium path. This allows us to treat debt as risk free.[14]

Utilities and budget constraints Agents $i = s, b$ maximize a time-separable utility function

$$U_i(c_i, h_i) = E_0 \sum_{t=0}^{\infty} \beta_i^t \, u(c_{s,t}, h_{s,t}) \tag{1.2}$$

where E_0 is the expectation operator at the the starting date $t = 0$. We consider period-by-period utility functions $u(c, h) : \mathbb{R}_{++} \times [0, 1] \to \mathbb{R}$ characterized by constant elasticity of substitution.

$$u(c, h) = \frac{\Psi(c, h)^{(1-\gamma)}}{1-\gamma}, \qquad \text{and} \qquad \Psi(c, h) = [\phi c^\rho + (1-\phi)h^\rho]^{\frac{1}{\rho}}$$

[13]Assuming default costs equal to zero, borrowers default in some successor node $\tilde{z} \in \sigma_t z^+$ iff

$$-mE[q(\sigma_t z^+)|\sigma_t]h_b(\sigma_t) + q(\tilde{z})h_b(\sigma_t) < 0,$$

That is, whenever the realized value of housing is smaller than the maximum amount promised. Since in any financial market equilibrium, house prices and - by the Inada conditions - h_b are strictly positive for a small enough m, this condition does not hold. As an alternative to a condition on m, we could just assume default costs are sufficiently high that it is never optimal for the borrowers to default.

[14]We evaluated the robustness of our results by replacing equation (1.1) by the following collateral requirement:

$$R_D(\sigma_t)d(\sigma_t) + m \cdot \min\left(q(\sigma_t z^+)\right) h_b(\sigma_t) \geq 0.$$

This is a tighter constraint and ensures that there is no default in equilibrium, independent of the value of m. While the qualitative implications remain unaffected, this specification implied slightly smaller quantitative effects on house prices and welfare. The intuition for the smaller quantitative effects is that leverage in states of high intermediation is lower compared to the benchmark model and the wealth distribution is therefore less sensitive to price changes. We stick to the collateral constraint as outlined in the main text because it has became standard in macroeconomic models with mortgage debt and thus increases the comparability of our results.

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Note that this class of preferences is strictly monotone, continuously differentiable, strictly concave, and satisfies the Inada conditions for both c_i and h_i.

At each node, the savers' budget constraint is given by

$$c_s(\sigma_t) + q(\sigma_t)h_s(\sigma_t) + s_s(\sigma_t) \leq y(\sigma_t) + s_s(\sigma_t^-)R(\sigma_t^-) + q(\sigma_t)h_s(\sigma_t^-) + \Upsilon(s^t). \tag{1.3}$$

The right hand-side is the savers' available income. It consists of the endowment of the perishable good $y(\sigma_t)$, the gross return on savings, and the housing stock carried over from the previous period. Finally, $\Upsilon(s^t)$ are resources that are redistributed in a lump-sum fashion from the financial sector to the households, of which savers receive a share n_s, representing their share in the population. The reason why we need this re-distribution will be explained in detail below.

Analogously, the borrowers' budget constraint reads as

$$c_b(\sigma_t) + q(\sigma_t)h_b(\sigma_t) + d_b(\sigma_t) \leq y(\sigma_t) + d(\sigma_t^-)R_D(\sigma_t^-) + q(\sigma_t)h_b(\sigma_t^-) + \Upsilon(s^t). \tag{1.4}$$

The right hand-side is the borrowers' available income. It consists of the endowment of the perishable good $y(\sigma_t)$, the value of housing stock net of the debt burden from the previous period plus resources being redistributed from the financial sector to the households, of which borrowers receive the amount $\Upsilon(s^t)$.

Financial Intermediaries. Intermediaries demand aggregate deposits $S(\sigma_t)$ and supply aggregate debt $D(\sigma_t)$. The real pay-offs for each unit lent are given by the real interest rates, $R_D(\sigma_t)$ and $R(\sigma_t)$, respectively. The collateral constraints and assumption 1 make sure that debt is risk free. The key distortion in the intermediation sector is similar to that in Cooper and Ejarque (2000).[15] We assume that in each node σ_t only a fraction of savings can be transformed into debt. This fraction is stochastic and depends on the realization of the current shock only. That is, $\theta(\sigma_t^- z) = \theta(z)$ and $\theta(z) : \mathcal{Z} \rightarrow (0,1]$ is a time-invariant function.

This exogenous financial shock represents a reduced form way to model the risk-bearing capacity of the financial sector. In particular, changes in the intermediation technology θ potentially reflect changes in the value of equity associated with a risky asset portfolio or changes in monitoring by the bank managers as a consequence of changes in risk aversion. Consequently, while we remain agnostic about the exact foundation of the θ, we point out that the observed variations in the spread series in the period 2005-2009 mainly reflect changes in the households' price for risk rather than changes in the default risk.[16]

[15] Another example for the inclusion of a supply-sided friction in the banking sector into an international macro model is Kalemli-Ozcan, Papaioannou, and Perri (2013).

[16] The inclusion of a more detailed micro-founded banking sector is an interesting avenue that we leave for future research.

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Financial intermediaries are otherwise risk neutral and maximize expected profits on their portfolio, that is,

$$\max_{D(\sigma_t),S(\sigma_t)\geq 0} R_D(\sigma_t)D(\sigma_t) - R(\sigma_t)S_i(\sigma_t) \tag{1.5}$$

subject to the constraint

$$D(\sigma_t) \leq \theta(\sigma_t)S(\sigma_t). \tag{1.6}$$

Because intermediaries operate in competitive markets with free entry, equilibrium interest rates are such that intermediaries make zero profits:

$$R_D(\sigma_t)\theta(\sigma_t) - R(\sigma_t) = 0. \tag{1.7}$$

This last relation implies that there is a spread between loan and deposit rates in this economy . In particular, the interest rate on debt is always at least as big as the interest rate on savings, or $R_D(\sigma_t) \geq R(\sigma_t)$.

Transfers from the Banking sector to the Household sector. Completing the description model, we specify the re-distribution function $\Upsilon(s^t)$. The intermediation process as outlined above implies an aggregate intermediation loss in terms of real resources that, in equilibrium, is given by $(1-\theta(\sigma_t))S(\sigma_t)$. This can be easily verified by combining the households budget constraints, using market clearing conditions in the debt and savings markets, and the zero profit condition of financial intermediaries. The aggregate resource constraint, then, reads as:

$$n_b c_b(\sigma_t) + n_s c_s(\sigma_t) + (1-\theta(\sigma_t))S(\sigma_t) = y(\sigma_t) + \Upsilon(s^t)$$

On the left hand side, we have the borrowers' and savers' consumption plus the resources 'eaten up' by the financial sector. On the right hand side we have aggregate income plus total transfers. In order to keep the intermediation process as a purely redistributive distortion, we choose $\Upsilon(s^t)$ such that all resources 'lost' in the intermediation sector are redistributed back to the agents, so that aggregate consumption is a function of aggregate income only. Therefore, aggregate transfers are defined as follows:

$$\Upsilon(s^t) \equiv (1-\theta(\sigma_t))S(\sigma_t) \tag{1.8}$$

We interpret this transfer as income generated by the intermediation sector that is redistributed back to the households because they are either the managers of the bank or the residual claimants on the portfolio revenues of the bank. The inclusion of the transfer function has two advantages. The first is that any effect of a θ shock on house prices and welfare comes through the effect on interest rates, and is not generated by an aggregate loss of resources. The second advantage is computational, as the re-distribution

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of resources makes sure that aggregate consumption is a function of aggregate endowment only, an essential requirement for the application of the concept of wealth recursive equilibria proposed by Kubler and Schmedders (2003) to our framework.

1.2.2 Financial Market Equilibrium with Intermediation and Houses as Collateral

The economy is a collection of period-by-period utility functions, impatience parameters, state-dependent endowments and state-dependent financial intermediation efficiency, aggregate transfers, transition probabilities, and the bankruptcy cost in case of default,

$$\mathcal{E} = \left(u, \left(\beta_i, y_i, h_i(\sigma_0^-)\right)_{i=b,s}, \theta, \Upsilon, \Pi, m\right).$$

Definition 1 *A financial markets equilibrium for an economy $\mathcal{E}$, initial housing stocks $(h_i(\sigma_0^-))_{i=b,s}$ and initial shock z_0 is a collection*

$$\Big((\bar{h}_b(\sigma_t), \bar{d}_b(\sigma_t), \bar{c}_b(\sigma_t)), (\bar{h}_s(\sigma_t), \bar{d}_s(\sigma_t), \bar{c}_s(\sigma_t)), (\bar{D}(\sigma_t), \bar{S}(\sigma_t)),$$
$$\bar{q}(\sigma_t), \bar{R}_D(\sigma_t), \bar{R}(\sigma_t), \bar{\Upsilon}(\sigma_t)\Big)_{\sigma_t \in \Sigma}$$

satisfying the following conditions:

(1) Markets clear for all $\sigma_t \in \Sigma$:

$$\begin{aligned} n_b\bar{h}_b(\sigma_t) + n_s\bar{h}_s(\sigma_t) &= 1 \\ \bar{D}(\sigma_t) + n_b\bar{d}_b(\sigma_t) &= 0 \\ \bar{S}(\sigma_t) - n_s\bar{s}_s(\sigma_t) &= 0 \end{aligned}$$

(2) For borrowers,

$$(\bar{h}_b(\sigma_t), \bar{d}_b(\sigma_i), \bar{c}_b(\sigma_t)) \in \arg \max_{c_b \geq 0, h_b \geq 0, d_b \leq 0} U_b(c_b, h_b)$$

such that for all $\sigma_t \in \Sigma$

$$\begin{gathered} c_b(\sigma_t) + \bar{q}(\sigma_t)h_b(\sigma_t) + d_b(\sigma_t) \leq y(\sigma_t) + d_b(\sigma_t^-)\bar{R}_D(\sigma_t^-) + \bar{q}(\sigma_t)h_b(\sigma_t^-) + \bar{\Upsilon}(\sigma_t) \\ \bar{R}_D(\sigma_t)d_b(\sigma_t) + m \cdot E[\bar{q}(\sigma_t z)|\sigma_t]h_b(\sigma_t) \geq 0 \end{gathered}$$

(3) For savers,

$$(\bar{h}_s(\sigma_t), \bar{s}_s(\sigma_i), \bar{c}_s(\sigma_t)) \in \arg \max_{c_s \geq 0, h_s \geq 0, s_s \geq 0} U_s(c_s, h_s)$$

such that for all $\sigma_t \in \Sigma$

$$c_s(\sigma_t) + \bar{q}(\sigma_t)h_s(\sigma_t) + s_s(\sigma_t) \leq y(\sigma_t) + s_s(\sigma_t^-)\bar{R}(\sigma_t^-) + \bar{q}(\sigma_t)h_s(\sigma_t^-) + \bar{\Upsilon}(\sigma_t)$$

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(4) For financial intermediaries

$$(\bar{D}(\sigma_t), \bar{S}(\sigma_i)) \in \arg\max_{D\geq 0, S\geq 0} \bar{R}_D(\sigma_t)D(\sigma_t) - \bar{R}(\sigma_t)S(\sigma_t)$$

such that for all $\sigma_t \in \Sigma$

$$D(\sigma_t) \leq \theta(\sigma_t)S(\sigma_t)$$

(5) Free entry for financial intermediaries

$$\bar{R}_D(\sigma_t)\bar{D}(\sigma_t) - \bar{R}(\sigma_t)\bar{S}(\sigma_t) = 0$$

(6) Per-capita transfers are given by

$$\bar{\Upsilon}(\sigma_t) = (1 - \theta(\sigma_t))\bar{S}(\sigma_t)$$

1.2.3 Wealth Recursive Equilibria

For the quantitative exercise, we define a wealth recursive formulation in the spirit of Kubler and Schmedders (2003). Since we have only two agents, the relative wealth of one agent, defined by a single value on the unit interval, uniquely define the complement of the other agent relative wealth; the borrowers' beginning-of-period wealth-share is :[17]

$$\omega_b(\sigma_t) = \frac{q(\sigma_t)h_b(\sigma_t^-) + R_D(\sigma_t^-)d(\sigma_t^-)}{q(\sigma_t)} \tag{1.9}$$

Note that the collateral constraints, the constraints on asset holdings, and the utility functions satisfying Inada-conditions, together with assumption 1, imply that the wealth share lies in the unit interval, $\omega_b \in [0, 1]$; by definition, $\omega_s = 1 - \omega_b$. The equilibrium policy function is then a function of the discrete exogenous state variable z and the financial wealth distribution is $\Omega = (\omega_b, 1 - \omega_b)$.

As we solve for an equilibrium numerically, we follow Kubler and Schmedders (2003) and compute ϵ-equilibria.[18] For the approximation of the equilibrium policy functions we adopt the time-iteration algorithm with linear interpolation proposed by Grill and Brumm (2010). That is, we approximate the equilibrium policy on a fine grid for the borrowers' wealth share. For points outside the grid we use linear piecewise interpolation. See appendix A.2 for a detailed description of the algorithm.

[17]Here, we used the market clearing conditions for the housing, debt, and savings markets and the fact that financial intermediaries make zero-profits in equilibrium, so that $h_b(\sigma_t^-) + h_s(\sigma_t^-) = 1$ and $R_D(\sigma_t^-)d_b(\sigma_t^-) + R(\sigma_t^-)s_s(\sigma_t^-) = 0$.

[18]For a definition and interpretation of ϵ-equilibria, we refer to the original text.

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1.3 Quantitative Analysis

This section studies the quantitative effects of the Great Recession on house prices and households' welfare. The Great Recession is modeled as contemporaneous negative shocks to both aggregate income and financial intermediation (mortgage rate spread). In this way, our simulation is driven by the empirical facts that motivated our research question. The next subsection outlines our calibration strategy. We then have a short section on the long-run stationary wealth distribution and we present our quantitative results on welfare effects.

1.3.1 Calibration

In the benchmark calibration, we assume an elasticity of substitution between houses and consumption equal to 1, so that $\rho = 0$. Risk aversion is set equal to $\gamma = 2$. These are standard values used in the literature. In general, it is not straightforward to calibrate these parameters as macro and micro evidence span a relatively large sets of parameter estimates. As in (Glover, Heathcote, Krueger, and Ríos-Rull, 2011), the risk aversion γ is the crucial parameter for the elasticity of house prices with respect to aggregate shocks. The elasticity of substitution between consumption and savings plays an important role for the elasticity of welfare gains/losses to changes in the wealth distribution. Therefore, in section 1.4, we provide a sensitivity analysis for different values of the risk aversion parameter and allow for some substitutability between housing and non-durable consumption as recently found by Bajari, Chan, Krueger, and Miller (forthcoming). Notice that one period in the model corresponds to one quarter in the data.

The parameter ϕ is the expenditure share of non-durable consumption. We pick the value to match the average housing wealth over GDP in the data during the period 1998-2007. For aggregate housing wealth, we used the sum of the value of owner occupied real estate of private households plus the residential housing wealth of non-financial non-corporate private business. The savers' discount factor β_s is set so that the average interest rate on savings in the model matches the average return on savings, equal to 1.5% during 1998 - 2007 (at annualized level). The borrowers' discount factor β_b and m are jointly calibrated to match the average wealth share of the borrowers and the leverage ratio of the borrowers. Since there is not necessarily a one-to-one mapping between the parameters and their targets, we follow an iterative procedure to find values for β_s, β_b, m and ϕ. That is, we first guess values for the parameters and then compare the computed moments to their counterparts in the data. If they do not match, we change the values and repeat until they do. The procedure leads to a quite satisfactorily match between model and

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Table 1.2: calibration

Parameter	Value	Model	Data Target	Source
Preferences				
γ	2			Benchmark value from literature
ρ	0			Benchmark value from literature
ϕ	0.97	196%	196%	Average housing value over GDP (annualized) 1998 - 2009
β_s	0.996	1.5%	1.5%	Average return on savings (annualized)
β_b	0.988	11.7%	11.3%	Borrowers' financial wealth share (SCFaverage 1998-2009)
m	0.5	45%	44.4%	Borrowers' leverage ratio (SCF average between 1998-2009)
Relative population size				
n_b	0.42	42%	42%	Share of borrowers (SCF average 1998-2009)
Intermediation shock				
π_H^θ	0.565		56.5%	Probability of low spreads during 1998-2009:II
ρ_θ	0.868	0.868	0.868	Autocorrelation of spreads during 1998-2009:II
θ_L	0.9985	1.8 %	1.75 %	Average spread during 1998-2009:II (annualized)
θ_H	0.99207	1.27 %	1.27 %	Standard deviation of spread during 1998-2009:II (annualized)
Income shock				
π_H^y	0.85	15%	15%	Probability of recession 1980- 2009:II (NBER dates)
π_{LL}^y	0.8	5 quarters	5 quarters	Average duration of recession (NBER dates) 1980- 2009:II
y_L	0.9572	5%	5%	Average Peak to trough drop in GDP 1980-2009:II
y_H	1.0076			Normalization $E(y) = 1$

data moments.[19]

[19]The variable definitions used to calculate the data moments are as close as possible to the definition of the model counterparts. For a detailed description of how we compute the relative wealth share and

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The relative population size of borrowers is set to 42%, corresponding to the fraction of borrowers in the SCF when using the weighted average share of households with a negative net asset position as defined in appendix A.1. This estimate is in line with the calibration in Iacoviello (2008).

The stochastic processes for the exogenous state variables y_t and θ_t are assumed to be independent. This is in line with the correlation in the data.[20] We assume that both aggregate income and the intermediation spread shock take two values each, that is $y_t = \{y_L, y_H\}$ and $\theta_t = \{\theta_L, \theta_H\}$. For the intermediation shock, we assume that the transition probabilities are given by:

$$\pi_{ij} = (1-\rho)\pi_j + \delta_{ij}\rho \qquad for\ i,j = H, L$$

where $\delta_{ij} = 1$ if $i = j$ and 0 otherwise; $\pi_j > 0$ is the unconditional probability of being in state j, and by definition we have $\sum_j \pi_j = 1$. The parameter ρ governs the persistence of the shock.[21] The unconditional probability of a high intermediation efficiency, $P(\theta = \theta_H)$, is set to 0.565, the fraction of quarters in which the U.S. experienced low spreads between 1998:I and 2009:II. We set $\theta_L = 0.99207$, $\theta_H = 0.9985$, and $\rho_\theta = 0.868$ so that we match the mean, standard deviation and the autocorrelation of the spreads in the data (for the data counterparts see table; for a description of the data see appendix A.1).

For the income shock, we choose y_H and y_L to match the mean, normalized to $E(y) = 1$, and an average peak-to-trough drop in GDP of 5% during a recession. The conditional probability of the low realization of y being in a recession today π^y_{LL} is chosen to match an average duration of a recession equal to five quarters. This is in line with the NBER recession dates between 1980:I and 2009:II. The transition probability of the high income realization conditional on high income today, $\pi^y_{HH} = 1 - (1 - \pi^y_{LL})\frac{1-\pi^y_H}{\pi^y_H}$, is obtained by setting the unconditional probability of a recession equal to 15% ($\pi_H = 0.85$). This is in line with NBER recession dates between 1980:I and 2009:II.

To summarize, the exogenous state space is then given by $\Sigma = \{(y_H, \theta_H),\ (y_L, \theta_H),\ (y_H, \theta_L),\ (y_L, \theta_L)\}$ and - given the assumption that income and intermediation processes are uncorrelated - the transition matrix for the exogenous process is just the Kronecker product of the individual transition probability matrices for the income shock and the intermediation shock. Table 1.2 summarizes the calibrated parameter values and the targets.

the leverage ratio in the data, see appendix A.1.

[20] We also conducted a VAR analysis for GDP growth and spreads for different lag-lengths and orderings and found no evidence for significant spillover terms and no contemporaneous correlations between GDP and mortgage spreads. Only in one specification (VAR of order two), the null of a Granger-causality of output growth on spreads is rejected, though the coefficients for individual lags of output were not significantly different from zero.

[21] See Backus, Gregory, and Zin (1989) and Mendoza (1991)

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1.3.2 Stationary wealth distribution

Figure 1.3: Wealth distribution

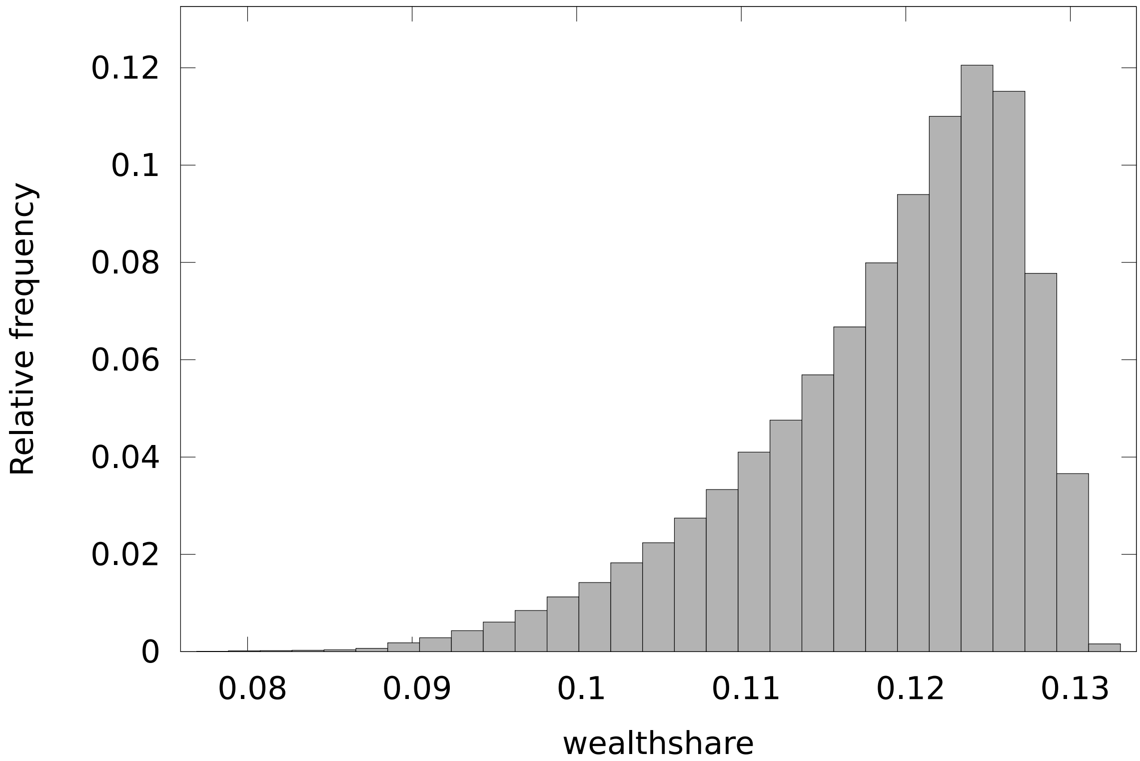

Figure 1.3 shows the long-run stationary wealth distribution simulated over one million time periods.[22] Recall that the wealth distribution across agents is entirely summarized by the borrowers' fraction of wealth ω_b. On average, the borrowers hold 11.7% of the total wealth of the economy (which is equal to the value of housing q). The distribution of the borrowers' wealth share is concentrated around the mean and has a spike to the right at around 12.6%, which correspond to states of the world when there is a long period of credit and income expansion. In these states, the borrowers' collateral constraint is binding and the interest rate on borrowing is relatively low; demand for housing is high and expected house prices are therefore high. This marginally relaxes the constraint, so that aggregate debt and savings are high. Because house prices are rising and borrowers are accumulating housing, their wealth share increases. Conversely, negative realizations of aggregate shocks make the borrowers' wealth share drop. We will explain these mechanisms in detail in the following section(s).

[22]Because of the simple persistence rule used to discretize the exogenous processes, the high number of simulation periods makes sure that the exogenous processes have the same stochastic properties as their data counterparts.

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1.3.3 Welfare effects in the Great Recession

We now turn to our main quantitative exercise, the estimation of welfare effects of the Great Recession. For this purpose we construct an event window around the Great Recession. We define the Great Recession as a state of the world with low income and high spreads that is preceded by a state of the world where income is high and spreads are low (i.e. intermediation is high). We then go along the equilibrium path of the simulated economy and select all sequences that match these criteria. In figure 1.4, we plot the average of selected realizations over all sequences including ten quarters preceding the crisis and ten quarters after the crisis. We compare the Great Recession to two counterfactual scenarios. First, we ask what would happen if spreads were low before and stayed low *during* the recession (this corresponds to the long dashed line in 1.4 which we label as *low-spreads* series). This experiment helps us to compare the welfare effects of a negative income shock when leverage is high or low before the shock realizes. Second, we look at a recession that occurs when spreads where already high before and during the crisis (short dashed line in figure 1.4 which we label as *high-spreads*). By comparing this scenario, with the Great Recession, we calculate the welfare effects of de-leveraging in the crisis.

Panel (a) and (b) show the evolution of income and mortgage spreads. In all scenarios, income first increases previous to the recession and then drops by 5 percent in period 0 when the recession hits. In the Great Recession, mortgage spread first decreases towards its lowest value in period -1 and then jump to 3.5 percent in period 0. In the *low-spreads* counterfactual scenario spreads decline and stay in their lowest realization in periods -1 and 0 and then return towards their long-run mean, around 1.75 percent per annum. Similarly in the *high-spreads* counterfactual scenario, spreads increase slowly previous to the recession, peaking at 3.5 percent p.a. in period 0 and then return slowly towards their long-run mean. From panel (c) it is evident that house-prices are clearly driven by aggregate income and not by mortgage spreads. Mortgage spreads, however, have an important impact on the borrowers' leverage ratio, defined as end-of-period leverage or $L_t^{EoP} = -\frac{d_t}{q_t h_{bt}}$; when spreads are low, borrowers leverage up by increasing their debt holdings faster than their housing wealth. This means they move towards the constraint. In our simulation, in the pre-crisis, leverage peaks at around 50 percent. When spreads increase in period 0, it becomes too costly for borrowers to roll-over their mortgages and de-leverage sharply so that the constraint gets slack. This is reflected by the multiplier associated with the collateral constraint that drops to zero. The time-path of leverage looks quiet different under the other two counterfactual scenarios. In the *low-spreads* case, borrowers stay leveraged also in period 0 and then de-leverage slowly following the path of spreads. In the *high-spreads* case, aggregate leverage is already low previous to the negative income shock and borrowers are pushed towards the collateral constraint

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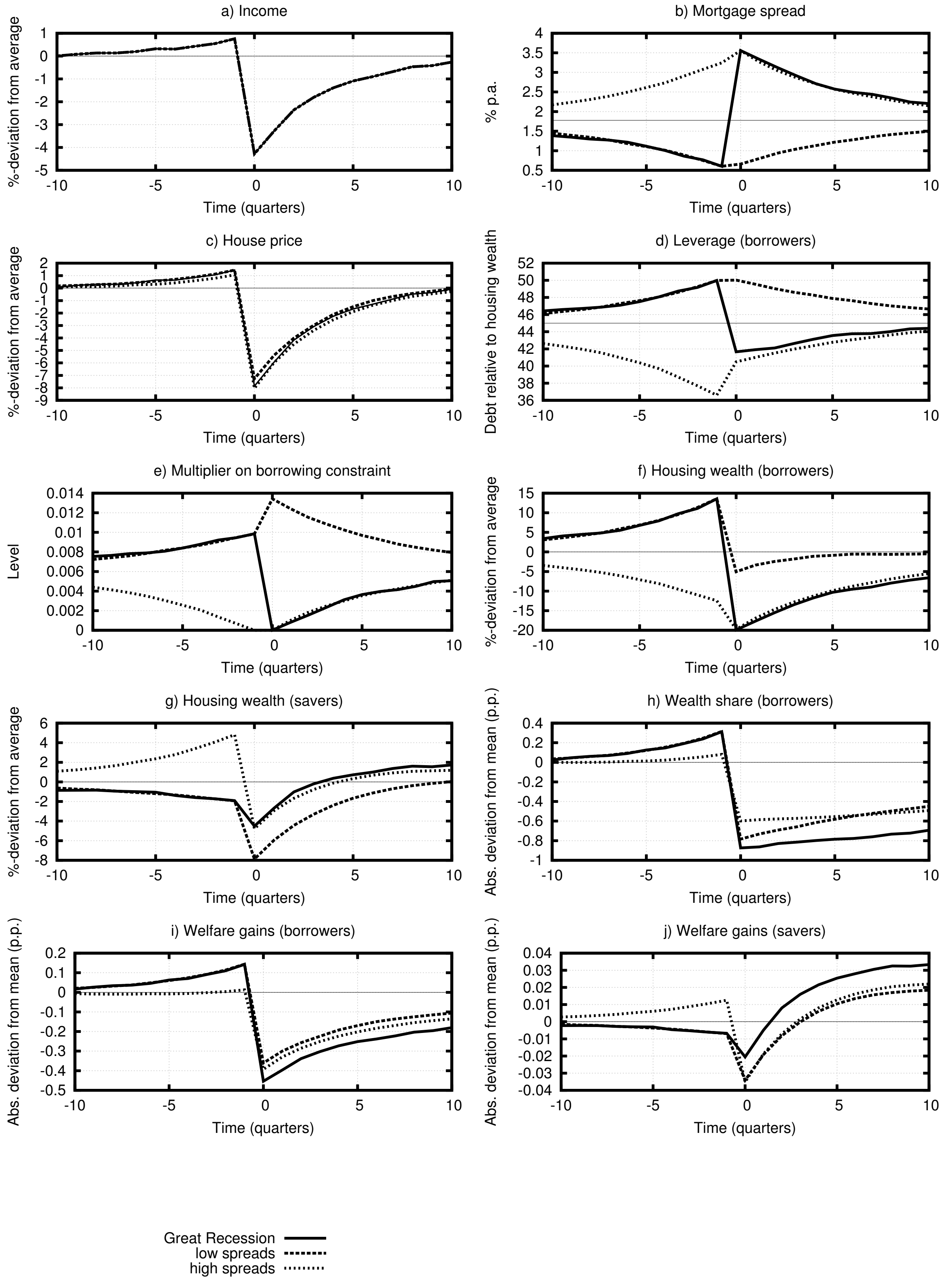
Figure 1.4: Great Recession (solid line) versus different intermediation regimes



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in period 0 when house prices fall. This is because borrowers search to smooth the recession by borrowing up to the limit (which is tighter because the house price drops in the recession). This is also reflected by the increase in the multiplier on the collateral constraint shown in panel (e). Therefore, shocks to financial intermediation affects the borrowers' leverage ratio through the relative price of debt (the mortgage spread). Panels (f) and (g) show the paths for housing wealth for borrowers and savers, respectively. This figures illustrate the following. If mortgage spreads would have stayed low during the recession (*low-spreads* case), borrowers would have lost less in terms of housing wealth than in the benchmark scenario, whereas savers would have lost more housing wealth. The movements in leverage and housing wealth are reflected by the evolution of borrowers' wealth share, shown in panel (h). In this panel the solid line shows drop much more than the the long-dashed line. Importantly the wealth share recovers much slower after the Great Recession compared to the case when mortgage spreads would have stayed low during the crisis. This means that borrowers negative wealth shock is quite persistent in the Great Recession. Finally, panels (i) and (j) show the corresponding welfare gains for the two type of households (in consumption equivalents relative to long-run expected utility, for a formal definition see next paragraph). Borrowers lose the most in the Great Recession while savers lose the least when compared to the other counterfactual scenarios. Note that only after two or three quarters, savers' expected life-time utility becomes positive and stays persistently above zero. This indicates substantial redistributive forces that is connected to the discussion about the borrowers' relative wealth share.

These findings are quantitatively formalized in table 1.3. The table compares the model predictions with the data (we observe the on-impact change in house price, the change in housing wealth for borrowers and savers in the period 2007-2009) and - in addition - shows the average change in borrowers' wealth share and the welfare gains/losses in the recession for the two types of households, denoted by λ_b and λ_s, respectively. We define welfare gains in two ways. First, we define welfare gains of the recession as the compensation that is needed to make agents indifferent between the expected life-time utility in period -1 (i.e. the quarter that precedes the recession) and expected life-time utility in period 0 (i.e. the quarter when the recession hits). Negative numbers therefore reflect welfare losses of the recession. We refer to these numbers as '*on-impact welfare gains*'. Second, we report welfare gains of the expected life time-utility that agents have 7 periods[23] after the recession relative to the average expected life-time utility, that is $\sum_{\sigma=1}^{4} \pi_\sigma V_i(\omega(\sigma), \sigma)$ for $i = b, s$.[24] Also in this case we report the welfare gains in percent of total consumption compensation that is needed to make agents indifferent between the two alternatives. We refer to this second type as '*welfare gains after 7 periods*'.

[23]The recent recession lastet 7 quarters according to NBER recession dates.

[24]The probability π_σ is the unconditional (or stationary) probability that state $\sigma \in \Sigma$ occurs.

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Table 1.3: Welfare effects of a recession (5 percent drop in income) for different spread regimes

	Δq	$\Delta(qh_b)$	$\Delta(qh_s)$	$\Delta\omega_b$	λ_b	λ_s
Data	-11	-16	-9	?	?	?
On impact, relative to pre-recession peak						
Great Recession	-9.18	-29.47	-2.65	-1.19	-0.60	-0.01
Low spreads	-8.59	-16.42	-6.07	-1.10	-0.50	-0.03
High spreads	-9.00	-8.34	-9.15	-0.68	-0.41	-0.05
After 7 periods, relative to long-run mean						
Great Recession	-1.29	-9.54	1.00	-0.78	-0.24	0.03
Low spreads	-1.01	-0.56	-1.13	-0.55	-0.15	0.01
High spreads	-1.37	-8.79	0.69	-0.54	-0.18	0.02

Notes: Column two shows the percentage change of the house price between date -1 and date 0, the period of the recession. Column three and four tabulate the percentage change in housing wealth between date -1 and 0 for borrowers and savers, respectively. Column four tabulates the absolute change of the borrowers' wealth share between date -1 and date 0 (in percentage points). Columns six and seven show the welfare gains of the recession in total consumption equivalents (relative to expected utility in period -1) for borrowers and savers, respectively. The Great recession is defined as a contemporaneous drop in income and financial intermediation (i.e. high spread) in period 0. The counterfactuals in row three (four) assume that financial intermediation is high (low) in both periods -1 and 0.

Based on figure 1.4 and table 1.3 we can summarize the following two key findings:

1. High leverage makes the borrowers' wealth share more sensitive to house price changes.
2. A negative intermediation shock, when coupled with a negative income shock, results in higher (smaller) welfare losses for borrowers (savers).

Result 1 says that the higher the leverage ratio in the economy when entering a recession, the more the wealth gets distributed away from borrowers to savers. In other words, a given house price drop due to an aggregate income shock leads to more bigger wealth losses for borrowers to savers when there is more leverage prior to the shock. If the economy is experiencing high intermediation efficiency previous to a recession, the leverage ratio of borrowers will be high. The borrowers' wealth share will then be very sensitive to price changes.

Result 2 deals with the second question raised in the introduction: whether a larger redistribution of wealth translates into more inequality in terms of welfare. We find that

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this crucially depends on whether the collateral constraint binds. That is, whether borrowers wish to stay up against the constraint, or move away from it. This result implies that the wealth loss from a recession only translates into a larger (smaller) welfare loss for borrowers (savers) when there is a simultaneous deterioration in the efficiency of financial intermediation. In particular, when spreads would have stayed low during the recession, shown in row three, the borrowers' welfare gain would have been 17 percent higher compared to the Great Recession. Savers would have lost three times more compared to the Great Recession. The intuition for both results is summarized in the following two paragraphs.

Intuition for Key Result 1 Let us now show the intuition behind these results graphically. To see the effects on the wealth distribution, we can rewrite the borrowers' wealth share in terms of the leverage ratio:

$$w_{b,t} = h_{b,t-1}(1 - L^{BoP}(q_t)) \tag{1.10}$$

where $L^{BoP}(q_t) = -\frac{R_{D,t-1}d_{b,t-1}}{q_t h_{b,t-1}}$ denotes the beginning of period leverage carried over from last period, evaluated at the house price of the current period.[25] Taking the total derivatives of the wealth share around $q_t = q_{t-1}$, one can see that the growth rate of the borrowers' wealth share is proportional to the growth rate of house prices and the proportionality factor is a function of leverage:

$$\frac{dw_{b,t}}{w_{b,t}} = \frac{L(q)}{1 - L(q)} \frac{dq}{q}.$$

If financial intermediation efficiency is low and spreads are high, leverage is likely to be small and a given drop in house prices translates into a smaller drop in wealth. In other words, when borrowers' leverage is high, any aggregate price drop makes borrowers - on impact - relatively poorer in terms of wealth.

Of course, the price today is an equilibrium outcome; that is, the pricing function depends on the state of the economy. We have no closed form solution for this pricing function but we can plot the equilibrium house prices as a function of the wealth share using the simulated economy. This function is - for any realization of the exogenous shock $z \in \mathcal{Z}$ - decreasing in w_b, or

$$q = Q(w_b, z) \qquad \frac{\partial Q}{\partial w_b} < 0. \tag{1.11}$$

[25] Note that by assumption 1, $L^{BoP}(q_t)$ is strictly smaller than one. This can be seen by the following. When leverage is high, most likely the collateral constraint is binding. Using the collateral constraint from last period and substituting it into the definition of beginning-of-period leverage, one obtains $m\frac{E_{t-1}(q_t)}{q_t}$. By assumption 1 and verified ex-post along the equilibrium path, this object is smaller than one.

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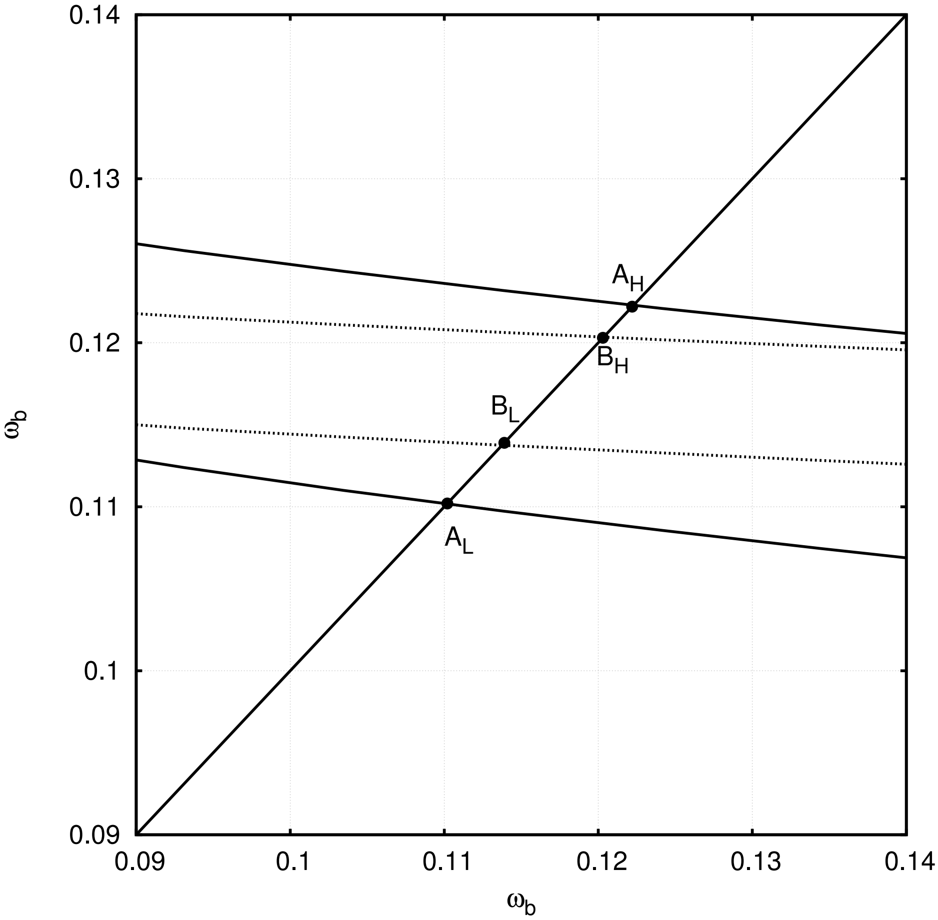
Given the promised value of previous-period debt, $R_{D,t-1}d_{b,t-1}$, and given the housing stock carried over from last period, $h_{b,t-1}$, the equilibrium wealth share in period t is implicitly defined by the solution to (1.10) and (1.11), or

$$w_{b,t} = h_{b,t-1}\left(1 + \frac{R_{D,t-1}d_{b,t-1}}{Q(w_{b,t}, z_t)h_{b,t-1}}\right) \tag{1.12}$$

Figure 1.5: Response of equilibrium wealth share to a negative income shock, for previously high (solid lines) versus low intermediation (dashed lines)



Notes: The figure plots the left-hand side (45 degree line) and the right hand side of equation (1.12) as a function of the borrowers' wealth share w_b and for different intermediation regimes. The solid lines show the right-hand side under the assumption that $h_{b,t-1}$ and $R_{D,t-1}d_{b,t-1}$ are relatively high (in absolute value) because of high financial intermediation. Given the assumption on debt and housing, point A_H materializes if income stays high whereas A_L is the wealth share when income drops to y_L. The dashed line shows the right-hand side under the assumption that $h_{b,t-1}$ and $R_{D,t-1}d_{b,t-1}$ are relatively low, that is for low intermediation. In this scenario, B_H is the wealth share that materializes when income stays high, whereas the wealth share drops to B_L when income falls to y_L

Figure 1.5 plots the left-hand side and right hand side of equation (1.12) as a function of the borrowers' wealth share w_b for different income realizations and for given assumptions

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on the level of debt and housing level. The solid line plots the right-hand side of equation 1.5 under the assumption that value of debt and housing stock in $t-1$ are relatively high (i.e. intermediation efficiency was high), while the dashed line assumes that debt and housing stock carried over from the previous period are low (i.e. financial efficiency was low).[26] When the previous period debt is high (solid line), the wealth share is more sensitive to exogenous shocks to income (drop from point A_H to A_L) compared to the case when debt carried over from last period is relatively low (drop from B_H to B_L). This illustrates the relationship between leverage and wealth dynamics during a recession: the effect comes from a different elasticity of wealth with respect to changes in prices which, in turn, depend on the aggregate state of financial intermediation.

Intuition for Key Result 2. Result (2) relates to combined income and negative intermediation shock. When house prices fall and there is a contemporaneous negative intermediation shock, borrowers face a higher interest rate on debt, which prevents them from rolling over the debt and moving away from the collateral constraint. This forces the borrowers to substantially decrease their stock of housing.

Figure 1.6 plots the borrowers' housing stock policy function for high and low intermediation efficiency (respectively solid and dashed line). Following the Great Recession, the relative wealth of the borrower drops. As financial intermediation also drops during the recession from high to low efficiency, the housing stock drops from A to C. This is a substantially larger drop than would have occurred had the efficiency of intermediation stayed high. In this case, for the same drop in wealth, the decrease of the housing stock would have been less sharp (from A to B). In other words, the elasticity of demand for housing with respect to income shocks depends on the efficiency of the financial intermediation sector.

Summary of the welfare effects. First, both agents lose in response to an aggregate negative income shock, and borrowers always lose more than savers because they are financially constrained and unable to cushion themselves from negative shocks. Second, while borrowers experience a welfare loss in the case of a negative financial intermediation shock, savers are virtually unaffected. Third, in the simulated recession, we observe that the borrowers' welfare loss is larger than the algebraic sum of the welfare losses in response to negative income and intermediation shocks in isolation. The opposite is true for the savers. This comes from a non-linearity in the reaction of consumption that comes when borrowers are forced to de-leverage and move away from the collateral constraint. In such

[26]We set the respective values for housing stock and debt equal to the average value in period -1 of the event window above for the respective intermediation regime.

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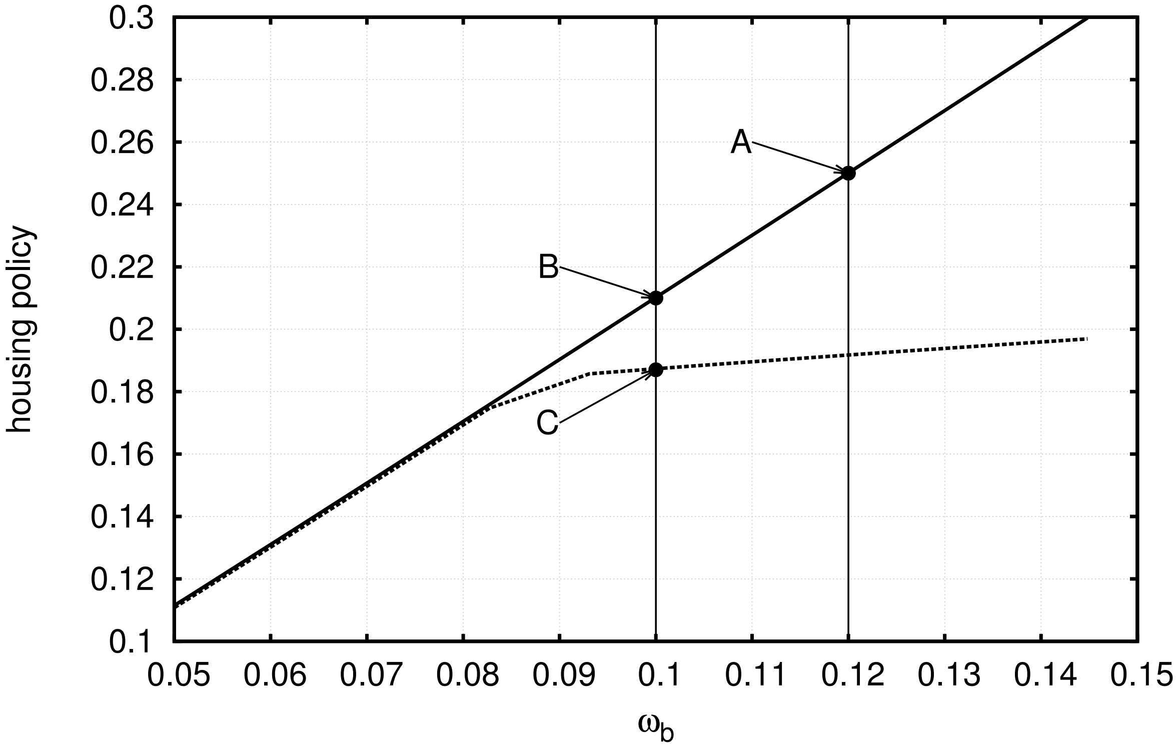
Figure 1.6: Equilibrium housing policy depends non-linear on wealth when financial intermediation efficiency changes from high to low



Notes: Solid line: housing policy as a function of the borrowers' wealth share, conditional on high financial intermediation efficiency. Dashed line: housing policy as a function of wealth, conditional on low financial efficiency. The vertical line intersecting at A is the borrowers' wealth share in a state with high income and high financial intermediation and the vertical line intersecting at B is borrowers' wealth share in the period when the Great Recession hits the economy.

a scenario savers can even gain from the joint income and intermediation shock (relative to an income shock alone) because they become relatively wealthier.

This set of results leads to the conclusion that, following the Great Recession, while both types of agents experienced a welfare loss, savers could cushion themselves from the negative impact of the negative aggregate shocks by substituting their savings for depreciated houses. This conclusion, while qualitatively comparable with the recent findings of Hur (2012), highlights a different mechanism. In this model, savers are able to cushion themselves from the negative effects of the Great Recession because of the asymmetric effects of financial intermediation shocks and the high level of leverage prior to the shock.

An important remark relating to the magnitudes of the obtained welfare estimates concerns the error analysis of our numerical algorithm. That is, if the mistakes agents make using our algorithm are larger (in consumption equivalents) than the calculated welfare gains/losses, these numbers would have no quantitative validity. We find that

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the maximum relative Euler Error of our approximation is 3e-5 (or -4.5 in log(10)-scale). This implies that an agent, using our approximation of the equilibrium policy functions, would lose 30 Dollars for each million spent. For details see appendix A.2.4. We therefore conclude that our quantitative findings are valid and quantitatively meaningful.

1.3.4 Always binding collateral constraint

We solve the model employing a global solution method rather than the more widely used log-linearization method. This is necessary in order to take into account the fact that the collateral constraint is not always binding, but comes at the cost of a more complex numerical implementation. In this section we show how large is the cost of assuming always binding constraints in this framework.

To this end, we solve an alternative specification of the model by forcing the borrowers to have an always-binding constraint. In this case, the leverage ratio of the economy is always equal to $m\frac{E_{t-1}q_t}{q_t}$, which therefore needs to be re-calibrated for this specification in order to match the leverage ratio we find in data. The results are summarized in table 1.4. Compared to the benchmark model, we find that in a version of the model with always-binding collateral constraints: (i) the quantitative effects on house prices are larger relative to the benchmark model for a negative financial intermediation shock; ii) in the Great Recession, the welfare losses for borrowers (savers) are smaller (higher) in absolute terms. To summarize, the borrowers' welfare loss is lower by 0.07 percentage points (in absolute terms), while the savers' lose 0.04 percentage points more when compared to the benchmark model. Most importantly, the non-linearity of previous-period leverage completely vanishes, as the borrowers' wealth losses and the agents' welfare gains are just the algebraic sum of the effects when the economy is hit with each shock separately.

The reason for these differences is that models with always binding constraint have the peculiarity of a constant elasticity of demand for debt with respect to changes in interest rate. In other words, following a spread shock, the borrowers' change in next period's debt has to be strictly proportional to the present discounted value of the drop in next period's housing wealth. When debt is costly, borrowers are prevented from moving away from the constraint. Aggregate debt moves less with respect to the benchmark case and this, in equilibrium, reduces the savers' ability to switch from savings to housing. This is the reason why house prices drop more in response to a negative intermediation shock. The elasticity of borrowers' wealth share to any given drop in house prices is always constant and given by $\frac{m^{AB}}{1-m^{AB}}$, where the superscript stands for 'always binding'. Note that, in order to match the average leverage ratio in the data, $m^{AB} = 0.45$, which is lower than $m = 0.5$ in the benchmark calibration. The elasticity of the borrowers' wealth share is

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Figure 1.7: Great Recession in benchmark model (solid line) versus always binding constraint (dashed line)

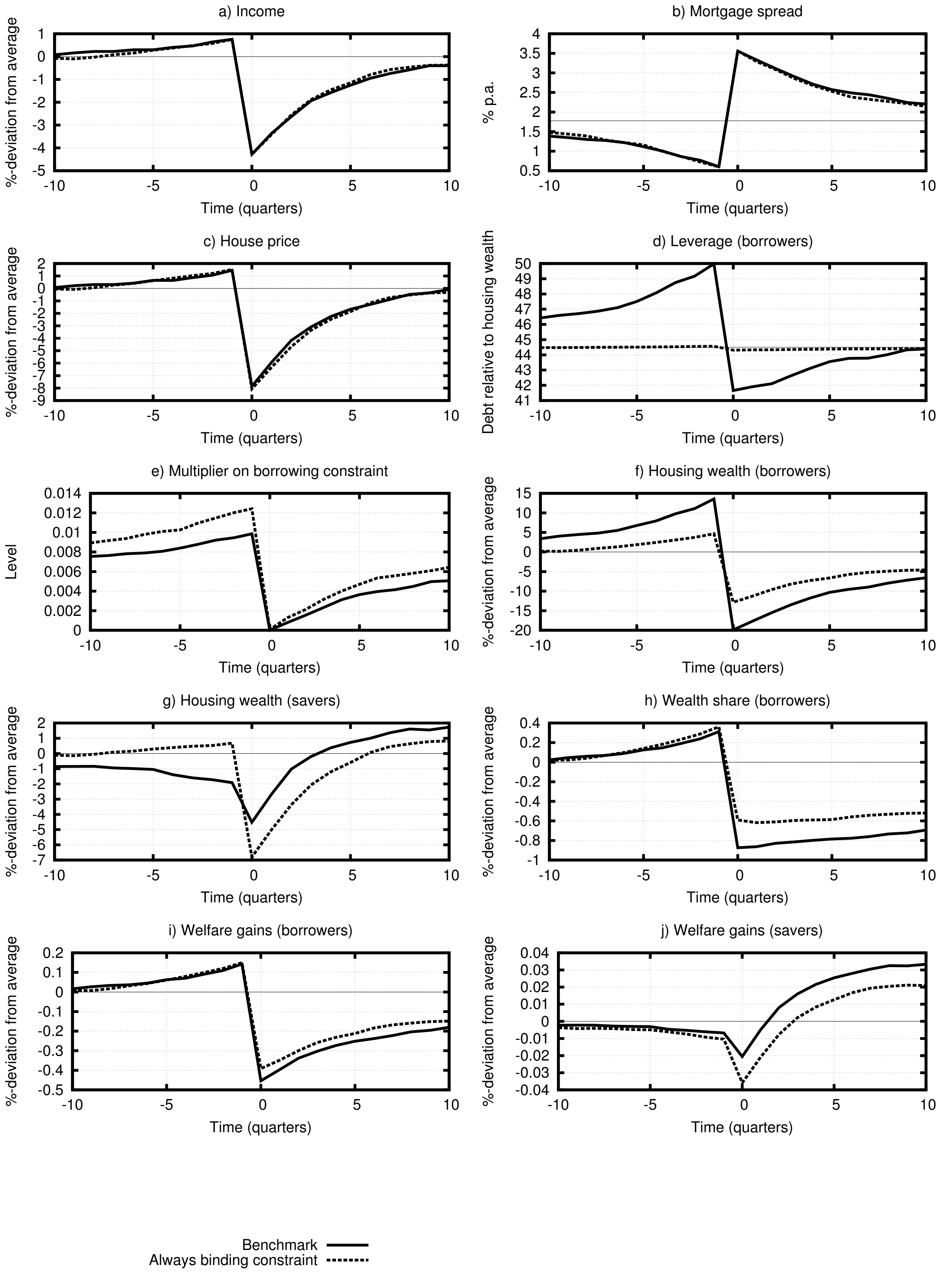

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Table 1.4: Always binding collateral constraint

	Δq	$\Delta(qh_b)$	$\Delta(qh_s)$	$\Delta\omega_b$	λ_b	λ_s
Data	-11	-16	-9	?	?	?
On impact, relative to pre-recession peak						
Great Recession	-9.44	-16.72	-7.43	-0.95	-0.54	-0.03
Low spreads	-8.65	-14.83	-6.94	-0.86	-0.46	-0.04
High spreads	-8.58	-15.17	-6.86	-0.88	-0.47	-0.04
After 7 periods, relative to long-run mean						
Great Recession	-1.16	-5.73	0.06	-0.56	-0.18	0.02
Low spreads	-1.21	-2.91	-0.75	-0.23	-0.09	-0.00
High spreads	-1.05	-7.83	0.76	-0.83	-0.25	0.03

Notes: Column two is the change in house prices between period -1 (the period just before the shock occurs) and period 7 (following the start of the recession). Column three shows borrowers' start-of-period leverage ratio, defined as $L_{b,t}^{BoP}$ in the period of the shock $t = 0$; note that this leverage ratio is a function of the price today only (variables with subscript $t-1$ are given numbers). Column four shows the corresponding change in borrowers' financial wealth share between period -1 and period 7. Column five reports the borrowers' end-of-period leverage ratio, defined as L_t^{EoP} after the Great Recession - in period $t = 7$. Columns six and seven show the welfare gains/losses of borrowers and savers, respectively. All numbers are in percent.

therefore constant, and is strictly less than one. This result suggests that the assumption of always-binding collateral constraints is not innocuous when making a welfare analysis.

1.4 Sensitivity Analysis

In this section we compare the quantitative implications of changing the elasticity of substitution between housing and non-durable consumption, and the coefficient of risk aversion. Note that, for all changes in these parameters, we re-calibrate the rest of the parameters that in order to match the targeted data moments. This allows us to compare the relative performance of each parameterization with the benchmark case.

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1.4.1 Elasticity of substitution between housing and non-durable consumption

Here we conduct a sensitivity analysis for one of the two parameters that we fixed in the benchmark calibration to unity: the elasticity of substitution between housing and non-durable consumption. Table 1.5 summarizes the quantitative findings for a higher level of substitutability between housing and non-durable consumption, setting $\rho = 1.25$.[27]

Table 1.5: Welfare effects in model with higher elasticity of substitution between housing and non-durable consumption

	Δq	$\Delta(qh_b)$	$\Delta(qh_s)$	$\Delta\omega_b$	λ_b	λ_s
Data	-11	-16	-9	?	?	?
On impact, relative to pre-recession peak						
Great Recession	-9.10	-38.87	-0.34	-1.19	-0.57	-0.02
Low spreads	-8.54	-17.83	-5.78	-1.12	-0.49	-0.03
High spreads	-9.09	-6.71	-9.51	-0.47	-0.35	-0.06
After 7 periods, relative to long-run mean						
Great Recession	-0.96	-12.92	1.88	-0.82	-0.23	0.03
Low spreads	-1.05	0.92	-1.52	-0.64	-0.16	0.02
High spreads	-1.52	-11.74	0.90	-0.34	-0.13	0.00

Table 1.5 shows that, with increasing substitutability between housing and non-durable consumption, house prices (and therefore wealth) react more strongly to an intermediation shock when compared to the benchmark case. This, like in the case with the always-binding constraint, results in a decreased elasticity of demand for debt with respect to changes in the interest rate for borrowing. In addition, the Great Recession leads to smaller (bigger) welfare losses for borrowers' (savers') in this calibration. Borrowers are hurt less because they substitute housing for non-durable consumption, which is less painful when these goods are substitutes. This is also the reason why there is less redistribution in terms of welfare from borrowers to savers. Though, in absolute terms, savers lose more. Nevertheless, the key findings relating to the role of leverage in wealth dynamics and the role of the intermediation shock in a recession are unchanged.

[27]This parameter value is taken from Piazzesi, Schneider, and Tuzel (2007), who consider a representative agent framework with housing; As mentioned earlier in the paper, an elasticity of substitution larger than one between housing and non-durable consumption has also recently been found by Bajari, Chan, Krueger, and Miller (forthcoming).

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1.4.2 Risk aversion

In this section we show quantitative analyses of Great Recession episodes for different values of the risk aversion parameter taken from the related literature. In particular, while the business cycle literature usually features a log-separable utility function with elasticity of substitution and risk aversion equal to unity, the macro-finance literature and recent contributions on the distributive effects of the Great Recession focus on a broader set of parameter values for risk aversion.[28] Table 1.6 summarizes the effects of the simulated Great Recession for the benchmark and other model specifications for different values of the risk aversion parameter.

Table 1.6: Welfare effects of the Great Recession, different risk aversion parameters

	Δq	$\Delta(qh_b)$	$\Delta(qh_s)$	$\Delta\omega_b$	λ_b	λ_s
Data	-11	-16	-9	?	?	?
On impact, relative to pre-recession peak						
$\gamma = 1$	-5.25	-22.52	-0.04	-0.60	-0.47	-0.04
$\gamma = 2$ (benchmark)	-9.18	-29.47	-2.65	-1.19	-0.60	-0.01
$\gamma = 3$	-12.80	-35.83	-4.84	-1.84	-0.73	0.02
$\gamma = 5$	-19.21	-37.31	-14.21	-2.33	-1.09	0.04
After 7 periods, relative to long-run mean						
$\gamma = 1$	-0.91	-6.95	0.72	-0.37	-0.15	0.01
$\gamma = 2$ (benchmark)	-1.29	-9.54	1.00	-0.78	-0.24	0.03
$\gamma = 3$	-1.38	-11.79	1.62	-1.21	-0.32	0.05
$\gamma = 5$	-1.40	-12.91	1.44	-1.52	-0.51	0.07

As in Glover, Heathcote, Krueger, and Ríos-Rull (2011), the higher is the coefficient of risk aversion, the higher is the negative impact of a recession on equilibrium aggregate house prices. The the observed drop in the house price during the Great Recession is consistent for a risk aversion parameter between 2 and 3. The welfare analysis also confirms that bigger wealth shocks (due to the drop in house prices) translate into larger negative welfare effects for borrowers. This effect is again amplified by financial intermediation

[28]Glover, Heathcote, Krueger, and Ríos-Rull (2011) set the risk aversion equal to 3 in the benchmark case, and then conduct a sensitivity analysis. They find that the magnitude of equilibrium price responses increase non-monotonically as risk aversion increases. Piazzesi, Schneider, and Tuzel (2007), in a capital asset pricing model with housing, find that a model featuring a higher level of risk aversion better performs in matching the moments of housing returns.

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shocks, which make it more difficult to smooth negative income shocks. In contrast, savers are more able to cushion themselves from the negative effects of the Great Recession. The intuition is the same as in the benchmark model. Following the reduction in aggregate debt, savers are able to reallocate their portfolios from savings towards housing (when it is relatively cheap). Consequently, the higher is the coefficient of risk aversion, the smaller are the overall welfare losses for savers.

1.5 Conclusions

Using a dynamic general equilibrium model calibrated to the US economy, we evaluate the quantitative effects of (i) aggregate income shocks and (ii) shocks to financial intermediation on house prices and on the welfare of two types of agents: leveraged agents (borrowers) and non-leveraged agents (savers).

The quantification of welfare costs associated with the US Great Recession along this cross-section complements recent contributions (Glover, Heathcote, Krueger, and Ríos-Rull, 2011; Hur, 2012) and adds a new mechanism stemming from shocks to the capital market. Our set-up is well suited for the evaluation of the welfare consequences of credit supply shocks in a recession, and complements other recent studies by exploring the effects of financial intermediation shocks in a model with endogenous collateral constraints.

We find that, following a shock modeled on the Great Recession, all the agents in the economy experience a welfare loss, and borrowers always lose more than savers. This finding comes from the fact that savers, being unconstrained, change their portfolio allocations and smooth the negative shock by buying the deflated asset (housing). We find that a financial intermediation shock that occurs in a recession forces borrowers to de-leverage, and amplifies the re-distribution from savers to borrowers, which translate in higher welfare losses for the latter.

Finally, we find that, in a model where borrowers are always borrowing constrained, the non-linearity in the amplification mechanism coming from the financial intermediation shock vanishes, and the effects on wealth and welfare are smaller.

We provide a number of sensitivity checks. While the redistributive effects (both in terms of financial wealth and welfare) between borrowers and savers are decreasing in the substitutability between housing and non-durable consumption, the drop in house prices is bigger when risk aversion is stronger, leading to a proportional increase in redistribution.

Although the paper focuses on the distributive effects of the Great Recession on borrowers and savers, we do not explicitly consider the possibility that borrowers can default on their

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debt obligations. While this could potentially benefit borrowers at the expense of their creditors, empirical evidence suggests that this feature of the U.S. Great Recession was restricted to a subset of borrowers, the sub-primers, who are not explicitly modeled here. Adding this third form of heterogeneity to the analysis is, in our opinion, an interesting avenue for future research.

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Chapter 2

Multinational Firms and Business Cycle Transmission

2.1 Introduction

A central question in international macroeconomics is how financial integration affects the international transmission of shocks. The literature on international business cycles has extensively analyzed the effects of financial assets, such as bonds or firm equity, on the long-run average implications of financial integration on international business cycles.[1] In terms of empirics, the literature typically focuses on the link between financial integration and output co-movement and is silent about investment co-movement.[2] This seems somewhat surprising because one robust prediction of the standard international real business cycle model is the strong negative co-movement of investment when financial markets become more integrated.[3] In terms of theory, the literature does not distinguish foreign direct investment (FDI) from other financial assets in regard of its consequences for international business cycle co-movement. Yet, FDI is special in that it involves technology flows within boundaries of multinational firms.[4] In this paper I attempt to make some progress along both the empirical and the theoretical dimension.

[1]Previous work includes Baxter and Crucini (1995), Kehoe and Perri (2002), Heathcote and Perri (2002). For a theoretical argument that the causality is reverse, see Heathcote and Perri (2004).

[2]Previous work includes Baxter and Kouparitsas (2005), Imbs (2006), and Hsu, Wu, and Yau (2011).

[3]See for example Backus, Kehoe, and Kydland (1992), Baxter and Crucini (1995), Kehoe and Perri (2002).

[4]See McGrattan and Prescott (2009), McGrattan and Prescott (2010), Ramondo and Rappoport (2010) and references therein.

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Empirical contribution. The main empirical contribution is to document that increases in bilateral FDI linkages are associated with more investment synchronization. This is a potential channel through which multinationals affect international business cycles that has been overlooked so far. In the benchmark regressions, I use a panel data-set of bilateral FDI linkages and data on GDP and investment synchronization for the G7 countries over the period 1991 - 2006. I explicitly exclude the recent crisis period because I do not want to have the results to be driven by a few observations.[5]

I also document that the link between FDI linkages and output synchronization is statistically indistinguishable from zero. This complements earlier work, as previous studies are inconclusive on this issue. The literature that focuses on FDI integration tipically finds a positive effect of financial integration on GDP synchronization.[6] On the other hand, a recent strand of literature focusing on banking integration suggests that the link between financial integration and business cycle co-movement is negative in normal times and positive in times of financial crisis.[7]

Theoretical contribution. From a theoretical perspective, my contribution is to build a dynamic stochastic equilibrium model of foreign direct investment. For this purpose, I embed technology capital and multinational production into a stochastic two-country real business cycle environment. The first objective of the model is to illustrate a concrete mechanism through which exogenous changes in FDI openness affect business cycle synchronization, in particular investment synchronization, and to study how this mechanism works both under shocks to country-specific aggregate productivity and shocks to multinational activity. The second purpose of the model is to conduct counter-factual analyses in order to shed light on what is driving the weak link between FDI integration and GDP synchronization as found in the data. Measured GDP is distorted because intangible investments by multinationals are expensed. Due to this mismeasurement, I show that the actual elasticity of output co-movement with respect to FDI openness is significantly higher than suggested by measured GDP. Third, I use the model to assess the risk sharing implications when FDI openness increases. I show that even when financial markets are complete, FDI integration reduces the consumption risk to which households are exposed. This reduction is strongest in the model with country-specific shocks only. Shocks to multinational activity, on the other side, mitigate the risk-reducing effect of more FDI because production in both countries is increasingly determined by

[5] In fact, business cycle correlations jumped up since the onset of the financial crisis in 2007, see Perri and Quadrini (2011).

[6] See, for example, Imbs (2004) and Hsu, Wu, and Yau (2011).

[7] See Kalemli-Ozcan, Papaioannou, and Peydró (2013) and Kalemli-Ozcan, Papaioannou, and Perri (2013).

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shocks to multinationals. This last result is the dynamic version of proposition 1 in Ramondo and Rappoport (2010) who consider a static multi-country environment.[8] To the best of my knowledge, I am the first one to make this point in a fully dynamic business cycle framework. This finding also complements Kalemli-Ozcan, Papaioannou, and Perri (2013) where the beneficial effects of more banking integration are mitigated by increasing exposure to global banking shocks.

The theory embeds features from the models studied in McGrattan and Prescott (2009) and Ramondo and Rappoport (2010) into a dynamic stochastic real business cycle framework. As in McGrattan and Prescott (2009), multinationals accumulate technology capital. This type of capital can be used simultaneously in different plants located both at home and abroad. Similar to Ramondo and Rappoport (2010), multinationals' productivity is subject to stochastic shocks and these shocks apply to all production units the multinational operates, both within and across country borders. By allowing for multinational-specific shocks, the multinationals itself act as a source of business cycles volatility - on top of affecting the propagation of country-specific shocks that originate in other sectors of the economy. In what follows, I consider different model versions (with and without multinational-specific shocks) that help to disentangle the role played by each model ingredient for the link between FDI openness and business cycle co-movement.

The model is also related to a growing strand of literature that stresses the importance of intangible capital for economic outcomes, such as asset prices (Eisfeldt and Papanikolaou, 2013), managerial compensation (Lustig, Syverson, and Van Nieuwerburgh, 2011), or the life-cycle of firms (Atkeson and Kehoe, 2005).[9] More closely related, Johri, Letendre, and Luo (2011) study the role of organizational capital for international investment co-movement; their model, however, abstracts from foreign direct investment and is therefore not suited to study the transition to FDI openness.

The paper is organized as follows. Section 2.2 reports the empirical methodology and the empirical results. Section 2.3 introduces the theoretical framework. Section 2.4 presents the quantitative results. Section 2.5 looks at the model implications for aggregate consumption risk and international correlations of consumption and hours worked. Section 2.6 concludes.

[8] The different layers of production in their economy is very similar to the one considered here. They do, however, abstract from investment in technology capital.

[9] These studies focus on organizational capital only whereas I adopt the broader definition of technology capital by McGrattan and Prescott (2009) that includes Brands, R&D, and organizational capital.

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2.2 Empirical results

2.2.1 Empirical specification

The empirical model is given by

$$synch^{a}_{i,j,t} = \theta_t + \gamma FDI_{i,j,t-1} + z'_{i,j,t}\beta + c_{i,j} + u_{i,j,t} \qquad \text{for} \qquad a = GDP, I. \tag{2.1}$$

where $synch^{a}_{i,j,t}$ is a time-varying bilateral measure reflecting the synchronization for growth in gross domestic product ($a = GDP$) and investment ($a = I$), respectively, between countries i and j in period t. One period in the regression setup is one year. The variable $FDI_{i,j,t-1}$ measures bilateral cross-border FDI positions between country i and j in the previous period (year) and $c_{i,j}$ is a country-pair specific unobserved heterogeneity that captures all time-invariant bilateral factors that affect both FDI integration as well as business cycle and investment synchronization.[10] I also include time dummies (θ_t) to account for shocks common to all countries. Following Kalemli-Ozcan, Papaioannou, and Perri (2013), in order to separate the relative importance of global and country-specific shocks, I also report results for specifications where only country-specific time trends (g_i and g_j) and where both aggregate and country-specific time trends are included:

$$synch^{a}_{i,j,t} = \theta_t + (g_i + g_j) \cdot t + \gamma FDI_{i,j,t-1} + z'_{i,j,t}\beta + c_{i,j} + u_{i,j,t} \qquad \text{for} \qquad a = GDP, I.$$

The vector $z'_{i,j,t}$ contains measure for trade linkages in and the product of the countries' income per capita and the countries' population. In addition, following the literature, I control for industrial specialization by taking the sum of each sectors' shares in total value added over all sectors.[11] All controls are lagged by one period to reduce the problem of potential endogeneity issues.

Following Kalemli-Ozcan, Papaioannou, and Perri (2013), I measure business cycle synchronization ($synch^{GDP}_{i,j,t}$) by the negative absolute distance in output growth rates between country i and country j in quarter t:

$$synch^{GDP}_{i,j,t} \equiv - \left| (\ln GDP_{i,t} - \ln GDP_{i,t-1}) - (\ln GDP_{j,t} - \ln GDP_{j,t-1}) \right|. \tag{2.2}$$

Analogously, cross-country investment synchronization in quarter t is defined as

$$synch^{I}_{i,j,t} \equiv - \left| (\ln I_{i,t} - \ln I_{i,t-1}) - (\ln I_{j,t} - \ln I_{j,t-1}) \right|. \tag{2.3}$$

[10] Other studies have stressed the importance of country-pair fixed effects, see Kalemli-Ozcan, Papaioannou, and Peydró (2013) and Kalemli-Ozcan, Papaioannou, and Perri (2013).

[11] See Imbs (2006).

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Table 2.1: Descriptive statistics

	N	Mean	Sd	Min	Max	p25	p50	p75	p95
G7 countries									
Pairwise corr. of hp-filtered GDP	336	0.373	0.434	-0.811	0.968	0.105	0.479	0.725	0.895
Pairwise corr. of hp-filtered Investm.	336	0.265	0.425	-0.848	0.969	-0.0418	0.297	0.607	0.883
Synch. of GDP	336	-2.317	1.192	-7.617	-0.332	-2.929	-2.076	-1.446	-0.872
Synch. of Investm.	336	-7.463	3.620	-25.12	-0.378	-9.335	-6.911	-4.845	-2.724
FDI/GDP	336	2.106	2.064	0.0729	11.45	0.504	1.524	3.147	6.449
FDI/total FDI	336	5.594	5.133	0.270	22.73	2.009	4.072	6.943	18.06
Trade/GDP	336	1.817	1.692	0.240	6.309	0.515	1.000	3.218	5.480
Trade/total trade	336	4.708	4.706	0.395	20.83	1.594	3.080	6.679	14.95
All country pairs									
Pairwise corr. of hp-filtered GDP	640	0.365	0.427	-0.811	0.968	0.0949	0.451	0.727	0.888
Pairwise corr. of hp-filtered Investm.	640	0.263	0.417	-0.848	0.969	-0.0326	0.302	0.583	0.881
Synch. of GDP	640	-2.501	1.427	-9.315	-0.187	-3.197	-2.129	-1.477	-0.898
Synch. of Investm.	640	-9.191	6.122	-42.54	-0.378	-11.13	-7.792	-5.090	-2.757
FDI/GDP	640	3.170	5.923	0.0466	54.70	0.475	1.492	3.491	10.77
FDI/total FDI	640	6.382	7.983	0.0762	50.92	1.400	4.072	8.050	19.14
Trade/GDP	640	1.721	1.749	0.0628	7.738	0.475	0.808	2.970	5.492
Trade/total trade	640	3.722	3.973	0.126	20.83	1.099	2.185	5.392	11.10

Notes: The table shows the descriptive statistics of the balanced sample for 40 country pairs from 1991 to 2006. The pairwise correlations of GDP and investment are the correlation of hp-filtered real GDP and real gross fixed capital formation, respectively, estimated using 20 quarter-rolling-windows. The GDP and investment synchronization indices are defined in equations (2.2) and equation (2.3), respectively. The indices are computed on a quarterly basis and then transformed into yearly observations by taking the average over four quarters. The synchronization indices are in percent (annualized); FDI and trade ratios are defined in equations (2.4) and (2.5), the unit is percent. For a data description and a list of country pairs included in the sample see appendix B.1.

The yearly estimates for aggregate and investment synchronization are obtained by averaging over the quarterly synchronization measures. As a robustness check, appendix B.2 reports estimates where the dependent variables are cross-country correlations of hp-filtered investment and GDP, respectively.

I measure cross-border FDI linkages in two ways. First, I use the sum of bilateral asset and liabilities between countries i and j over the sum of the two countries' GDP in each year:[12]

$$\left(\frac{FDI}{GDP}\right)_{i,j,t} \equiv \frac{FDIA_{i,j,t} + FDIL_{i,j,t} + FDIA_{j,i,t} + FDIL_{j,i,t}}{GDP_{i,t} + GDP_{j,t}}. \tag{2.4}$$

Second, I use bilateral FDI assets and liabilities divided by the sum of total FDI assets and liabilities of the two countries:[13]

$$\left(\frac{FDI}{TotFDI}\right)_{i,j,t} \equiv \frac{FDIA_{i,j,t} + FDIL_{i,j,t} + FDIA_{j,i,t} + FDIL_{j,i,t}}{FDIA_{i,t} + FDIL_{i,t} + FDIA_{j,t} + FDIL_{j,t}}. \tag{2.5}$$

The sample for the empirical analysis in the main text consists of the 21 G7 country

[12]See Kalemli-Ozcan, Papaioannou, and Perri (2013).

[13]I also normalized bilateral FDI positions using total foreign assets and liabilities, finding similar results.

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pairs for the years between 1991 and 2006. For a data description see Appendix B.1. The appendix also confirms the estimation results for a wider set of country pairs, using a balanced panel with 40 country pairs from 1991 to 2006.[14] Table 2.1 reports the descriptive statistics for the relevant variables in the sample.

2.2.2 FDI linkages and investment synchronization

This section reports the findings on the relation between FDI integration and international investment synchronization. Table 2.2 reports the benchmark estimates on the relation between FDI integration and investment synchronization for the G7 countries for the period 1991- 2006. The specification in column (1) controls for country-pair fixed effects and country specific time trends. The coefficient is positive and statistically different from zero. That is, conditional on country specific shocks, within country-pair increases in FDI integration are associated with more synchronized investment. In column (2), I include time fixed-effects to account for common global shocks, while column (3) reports results with time fixed-effects and country-specific time trends. In all specifications but specification (2), the coefficient on FDI integration is positive and statistically different from zero. In column (4), I control for bilateral trade linkages.[15] The coefficient on goods trade is positive and similar in magnitude as the coefficient on FDI integration. Yet we cannot reject the Null of a zero coefficient. Most importantly, when controlling for goods trade does not affect the coefficient on FDI integration.[16]

To get a sense for the magnitudes, note that FDI linkages are expressed in logs and investment synchronization is in percentage points, hence the coefficients reflect semi-elasticities. The coefficient in column (3) implies that a doubling in bilateral integration (e.g., when moving from the 50 percent percentile to the 75 percent percentile of FDI linkages) is associated with an average increase in investment synchronization of 1.8 percentage points. Given the median investment synchronization is equal to -7.5 percent for the G7 countries these are economically large effects.

Columns (5) to (8) report the results using the alternative FDI integration index as

[14]The reason for using the restricted sample is data availability. For these country pairs there are no missing values for bilateral FDI positions and we have a balanced sample. Using the full (unbalanced) sample with 18 countries from 1985 to 2006 does not alter the main conclusions. Appendix B.2 reports additional estimation results and robustness checks.

[15]Similar to FDI linkages, bilateral trade is defined as the log of the sum of bilateral trade flows divided by the sum of the countries' GDP.

[16]Earlier work (e.g. Frankel and Rose (1998) or Kose and Yi (2006)) showed the importance of trade for aggregate business cycle co-movement. The positive point estimates suggest the existence of some complementarity between FDI and trade. Yet, the trade linkages are only moving slowly over time, so there might be too little within-country correlation to pick up significant effects.

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Table 2.2: Bilateral FDI Linkages and Investment synchronization, G7 country pairs

Dependent Variable: Investment growth synchronization (annualized)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
FDI/GDP	1.538*	0.537	1.752**	1.716**				
	(1.85)	(1.24)	(2.62)	(2.78)				
Trade/GDP				0.525				
				(0.25)				
FDI/Total FDI					0.492	0.635	1.430**	1.544***
					(0.59)	(1.33)	(2.42)	(2.90)
Trade/Total Trade								-1.315
								(-0.61)
Country-pair fixed	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time fixed	No	Yes	Yes	Yes	No	Yes	Yes	Yes
Country trends	Yes	No	Yes	Yes	Yes	No	Yes	Yes
R-squared (within)	0.160	0.357	0.387	0.387	0.154	0.358	0.385	0.385
Observations	315	315	315	315	315	315	315	315

Notes: The table reports panel (country-pair) fixed-effect coefficients estimated over the period 1991 to 2006 for the 21 G7 country pairs. The dependent variable is minus one times the absolute value of the difference in quarterly growth rate of aggregate investment between country i and j in year t (the yearly estimate is obtained by averaging over the respective four quarterly estimates). In columns (1) - (4) FDI integration is measured by the log of the share of the stock of bilateral Foreign Direct Investment Assets and Liabilities between countries i and j in the previous year relative to the sum of the two countries' GDP in the previous year (denoted FDI/GDP). In columns (5) - (8) FDI integration is measured by the log of the share of the stock of bilateral Foreign Direct Investment Assets and Liabilities between countries i and j in the previous year relative to the sum of the two countries' total FDI Assets and FDI Liabilities in the entire world in the previous year (denoted FDI/Total FDI). All specifications also include the log of the two countries' per capita GDP, the log of the product of the two countries' population, and the log of the industrial specialization index as defined in the appendix; all controls are included with one period lag. The specification in (4) includes the log of the share of bilateral export and import flows between countries i and j in the previous year relative to the sum of the two countries' GDP in the previous year (Trade/GDP). The specification in (8) includes the log of the share of bilateral export and import flows between countries i and j in the previous year relative to the sum of the two countries' total exports and imports in the previous year (Trade/Total Trade). The specifications in columns (1) and (5) include country-specific linear time-trends. The specifications in columns (2) and (6) include time fixed-effects. The specifications in columns (3),(4),(7), and (8) include time fixed-effects and country-specific linear time-trends. Standard errors adjusted for panel (country-pair) specific auto-correlation and heteroskedasticity and corresponding t-statistics are reported below the point estimates. A † denotes significance at the 85% confidence level, * denotes significance at the 90% confidence level, ** denotes significance at the 95% confidence level, *** denotes significance at the 99% confidence level. For a detailed data description see appendix B.1.

defined in equation (2.5). The results are similar to the ones presented in columns (1) to (4). More FDI linkages are associated with higher investment synchronization; the point estimates are somewhat lower than the ones in specifications (1) to (4).

The estimated coefficients are robust to a number of robustness checks. In particular, using the full unbalanced sample for all available country pairs from 1985 to 2006, restricting the sample to the balanced sample of 40 country-pairs between 1991 - 2006, or using as a dependent variable the cross-country correlation of hp-filtered investment does not change the main results: higher FDI linkages are associated with more investment synchronization and the effect is economically large. Appendix B.2 contains more details

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Table 2.3: Bilateral FDI Linkages and GDP synchronization, G7 country pairs

Dependent Variable: GDP growth synchronization (annualized)								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
FDI/GDP	0.617**	0.108	0.317	0.346				
	(2.26)	(0.69)	(1.28)	(1.41)				
Trade/GDP				-0.421				
				(-0.74)				
FDI/Total FDI					$0.447^{\dagger}$	0.0763	0.207	0.222
					(1.57)	(0.47)	(0.80)	(0.86)
Trade/Total Trade								-0.172
								(-0.29)
Country-pair fixed	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time fixed	No	Yes	Yes	Yes	No	Yes	Yes	Yes
Country trends	Yes	No	Yes	Yes	Yes	No	Yes	Yes
R-squared (within)	0.179	0.343	0.365	0.366	0.172	0.343	0.363	0.363
Observations	315	315	315	315	315	315	315	315

Notes: The table reports panel (country-pair) fixed-effect coefficients estimated over the period 1991 to 2006 for the 21 G7 country pairs. The dependent variable is minus one times the absolute value of the difference in quarterly growth rate of real GDP between country i and j in year t (the yearly estimate is obtained by averaging over the respective four quarterly estimates). In columns (1) - (4) FDI integration is measured by the log of the share of the stock of bilateral Foreign Direct Investment Assets and Liabilities between countries i and j in the previous year relative to the sum of the two countries' GDP in the previous year (denoted FDI/GDP). In columns (5) - (8) FDI integration is measured by the log of the share of the stock of bilateral Foreign Direct Investment Assets and Liabilities between countries i and j in the previous year relative to the sum of the two countries' total FDI Assets and FDI Liabilities in the entire world in the previous year (denoted FDI/Total FDI). All specifications also include the log of the two countries' per capita GDP, the log of the product of the two countries' population, and the log of the industrial specialization index as defined in the appendix; all controls are included with one period lag. The specification in (4) includes the log of the share of bilateral export and import flows between countries i and j in the previous year relative to the sum of the two countries' GDP in the previous year (Trade/GDP). The specification in (8) includes the log of the share of bilateral export and import flows between countries i and j in the previous year relative to the sum of the two countries' total exports and imports in the previous year (Trade/Total Trade). The specifications in columns (1) and (5) include country-specific linear time-trends. The specifications in columns (2) and (6) include time fixed-effects. The specifications in columns (3),(4),(7), and (8) include time fixed-effects and country-specific linear time-trends. Standard errors adjusted for panel (country-pair) specific auto-correlation and heteroskedasticity and corresponding t-statistics are reported below the point estimates. A † denotes significance at the 85% confidence level, * denotes significance at the 90% confidence level, ** denotes significance at the 95% confidence level, *** denotes significance at the 99% confidence level. For a detailed data description see appendix B.1.

on these robustness checks.

2.2.3 FDI Linkages and GDP synchronization

In this section, I present the results of the benchmark estimations for the relation between integration and business cycle correlation. Table 2.3 reports the benchmark estimates on the effect of FDI linkages on GDP synchronization in the period 1991- 2006. In column (1), controlling for country-pair fixed effects and country specific time trends, the coefficient is positive and statistically different from zero. That is, conditional on

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country specific shocks, within country-pair increases in FDI integration are associated with more synchronized investment. However, this result is not robust to the inclusion of an aggregate time trend. In column (2), I include time fixed-effects to account for common global shocks, while column (3) reports results with both time fixed-effects and country-specific time trends. In both specifications, the coefficient on FDI integration remains positive but is statistically indistinguishable from zero. Specification (4) controls for bilateral trade linkages. The coefficient on goods trade is positive and bigger in size than the coefficient on FDI integration. Yet we cannot reject the Null of a zero coefficient.[17] From the table also emerges that controlling for goods trade does not affect the finding of a quantitative small and statistical insignificant link between FDI integration and GDP synchronization.

Columns (5) to (8) report the results using the alternative FDI integration index as defined in equation (2.5). The results are similar to the ones in columns (1) to (4), except that in specification (8) the coefficient on trade linkages becomes statistically significant at the 10 percent level. The results regarding FDI integration and GDP synchronization remain unchanged: there is no statistical significant link between FDI linkages and GDP synchronization.

2.3 A model of international business cycles with foreign direct investment

In this section, I develop a model of international business cycles where multinationals accumulate technology capital and engage in FDI. Technology capital is firm-specific and can be simultaneously used in multiple plants in locations at home and abroad.[18] The plants operatad by multinationals thus produce all with the same technology capital. There are two types of shocks causing economic fluctuations: a standard country-specific productivity shock and a shock that is multinational-specific, affecting the efficiency of the existing technology capital. This multinational-specific shock therefore affects both the returns on domestic and on foreign investment.

The model serves three purposes. The first is to precisely lay out a causal link between FDI openness and international investment synchronization. The empirical section documents a relationship between the two, but does not speak about the underlying mech-

[17] A reason for this finding could also be reverse causality: less correlated country pairs engage in more FDI. In this case, the presented coefficients presented here are lower bounds as this argument describes a downward bias for the un-instrumented estimates. For the theoretical argument, see (Heathcote and Perri, 2004).

[18] For the concept of locations in this context refer to McGrattan and Prescott (2009).

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anism and the direction of causation. I will use the model to derive quantitative results that show how the empirical findings are indeed consistent with the hypothesis that FDI openness has significant effects on investment synchronization. The second purpose of the model is to shed light on the weak link between FDI openness and GDP synchronization, as documented in the empirical section. For this purpose, I measure GDP in the model in the same way as in national accounts data where investments in intangible capital are expensed. With the quantitative results of the model, I show that FDI openness has indeed weak effects on business cycle synchronization when using measured GDP as a proxy for aggregate activity. Third, and relatedly, I use the model to conduct a counter-factual analysis to show that the relation between FDI openness and business cycle synchronization is significantly stronger when aggregate output is measured correctly.

The framework combines earlier work from Ramondo and Rappoport (2010) and McGrattan and Prescott (2009) to incorporate multinational production into an international business cycles set-up. The main innovation is that I consider both a stochastic environment and allow for an explicit role for FDI; as such, the set-up is well-suited to analyze the effects of cross-border FDI integration on investment synchronization. As will be shown below, a key ingredient of the model is the accumulation of technology capital.

2.3.1 The economy

I consider a two-countries, two-sectors, two-goods world. In each country (foreign variables are denoted by an asterisk), there are households of equal mass normalized to unity that consume a tradable final consumption good and supply labor to firms. Firms in the final good sector buy intermediate inputs from intermediate good firms, hire labor, accumulate physical capital and pay wages and dividends to domestic households. Physical capital and labor are not mobile across countries but across sectors. The intermediate good is not tradable across countries and producers in this sector buy differentiated goods from domestic and foreign firms, labelled multinationals. Multinationals can accumulate physical capital in both countries and set up production units both at home and abroad through which they serve the foreign intermediate goods market. Multinationals pay dividends to their owners, domestic multinationals are entirely owned by domestic households and foreign multinationals are entirely owned by foreign households.[19] In addition to physical capital, multinationals accumulate technology capital. Technology capital is

[19]I exclude that firm shares are traded. Because financial markets are complete this is without loss of generality.

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firm-specific and can be used in multiple locations in both countries at the same time. For international financial markets, I consider two scenarios: one in which international financial markets are complete in the sense that households have access to a full set of state-contingent securities that can be traded internationally. The other scenario is one in which households cannot trade any international assets and just receive labor income and dividends from domestic firms and multinationals.

Time and uncertainty. Time is discrete and denoted by $t = 1, 2, \ldots$. In each period t the economy experiences one event $s_t \in S$ where S is a possibly infinite set. I denote by s^t the history of events up to and including date t. The probability at date 0 of any particular history s^t is given by $\pi(s^t)$. For the sake of readability (and with some abuse of notation), I will drop the explicit reference to histories and states most of the time when there is no room for confusion; I will use the subscript t instead to refer both to the time period and histories.

Households. Households supply labor and the total supply of time is normalized to $\bar{L}$; households derive utility from consumption of the perishable good C_t and from leisure $\bar{L} - L_t$. Households maximize the expected discounted sum of future period utilities given by

$$E \sum_{t=0}^{\infty} \beta^t U(C_t, \bar{L} - L_t)$$

where E represents expectations across all possible states of the world, C_t denotes consumption, L_t is labor effort, $0 < \beta < 1$ is the discount factor, and the period-by-period utility function is given by $U(C_t, \bar{L} - L_t) = \log(C_t) + \alpha \log(\bar{L} - L_t)$. Given aggregate wages w_t, households receive labor income $w_t L_t$ and dividend payments from domestic tradable good firms d_{Tt} and from multinationals d_{Mt}, respectively.

International financial markets. I consider two versions of the model, one with complete international financial markets and one with financial autarky, in the sense that households cannot trade any international assets.

1. In the **complete** financial markets scenario, households have available a complete set of Arrow securities. Let $B_t(s^t, s_{t+1})$ be the quantity of bonds purchased by the home households at time t after history s^t that pay one unit of the consumption good in $t + 1$ if and only if the state of the world economy in $t + 1$ is equal to s_{t+1}. Let $q_t(s^t, s_{t+1})$ be the price of such a bond. Under complete international financial markets, the budget constraint for the representative household in the home country

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is

$$C_t + \sum_{s_{t+1}} q_t(s^t, s_{t+1}) B_t(s^t, s_{t+1}) = w_t L_t + d_{Tt} + d_{Mt} + B(s^{t-1}, s_t) \tag{2.6}$$

and the budget constraint for foreign households is analogously defined.

2. Under **financial autarky**, households are not allowed to trade any financial asset across country borders. In this model version, the budget constraint for the representative household in the home country is

$$C_t = w_t L_t + d_{Tt} + d_{Mt}, \tag{2.7}$$

analogous for the foreign households.

Firms in the tradable goods sector. The tradable consumption good is produced under perfect competition with a constant returns to scale technology that combines labor (l_{Tt}), capital (k_{Tt}) and the composite intermediate good (X_t). Production in the this sector is subject to stochastic and country-specific productivity shocks a_t and a_t^*. Firms' production function is given by

$$Y_t = e^{a_t} \left(k_{Tt}^{\theta} l_{Tt}^{1-\theta} \right)^{\nu} X_t^{1-\nu}, \tag{2.8}$$

where $0 < \nu < 1$. Final good firms purchase X_t units of the intermediate good from competitive intermediate good producers at a unit price P_t, where I normalized the price of the tradable good to one. Firms' dividends are thus given by

$$d_{Tt} = e^{a_t} \left(k_{1t}^{\theta} l_{1t}^{1-\theta} \right)^{\nu} X_t^{1-\nu} - P_t X_t - w_t l_{Tt} - i_{Tt} \tag{2.9}$$

where i_{1t} represents investment in physical capital. The capital stock evolves according to

$$k_{Tt+1} = (1-\delta) k_{Tt} + \left[\frac{\chi_1 \left(\frac{i_{Tt}}{k_{Tt}} \right)^{1-\psi}}{1-\psi} + \chi_2 \right] k_{Tt} \tag{2.10}$$

where δ is the depreciation rate, ψ determines the sensitivity of the cost to investment, and the parameters χ_1 and χ_2 are set by imposing steady state targets.[20] I assume that the productivity shocks follow a bivariate auto-regressive process

$$\begin{bmatrix} a_t \\ a_t^* \end{bmatrix} = \Lambda_a \begin{bmatrix} a_{t-1} \\ a_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t^a \\ \varepsilon_t^{a^*} \end{bmatrix} \tag{2.11}$$

[20]This functional form is widely used in the literature, see for example Quadrini and Jermann (2012). The parameters are chosen such that the depreciation rate is equal to δ and that the derivative of capital with respect to investment is equal to one.

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where Λ_a is a 2×2 matrix and $[\varepsilon_t^a, \varepsilon_t^{a^*}]'$ is a vector of i.i.d. random variables with mean 0, standard deviation σ_a and correlation ρ_ε^a. The problem of domestic tradable goods firms is then

$$\max E \sum_t^\infty Q_t d_{Tt}$$

subject to (2.9), (2.10), k_{10} given, where $Q_t = \beta^t U_c(C_t, L_t)$ is the marginal utility of period t consumption of domestic consumers who are the owners of the firm. The problem of tradable good firms in the foreign country is analogous.

Intermediate good producers. Intermediate good producers buy non-tradables produced by multinationals and sell the bundled good Y_{It} at price P_t to final good producers. The index Y_{It} aggregates a continuum of intermediate goods with a constant elasticity of substitution $\frac{1}{1-\eta}$. As discussed in more detail below, I assume that there are only two types of firms in each country: (i) domestic multinationals and (ii) foreign multinationals. Given this assumption, intermediate good producers' output reads as

$$Y_{It} = \left[\int x_t(i)^\eta \right]^{\frac{1}{\eta}} \tag{2.12}$$

where $x_t(i)$ denote the intermediate good producers' demand for goods produced by domestic and foreign multinationals, respectively. The implied demand functions are given by

$$x_t(i) = (P_t / p_t(i))^{\frac{1}{1-\eta}}. \tag{2.13}$$

The main effect of adding imperfect competition to the model is that it scales up the amount of variable profits in the economy; hence, it scales up the size of the payments owners receive from technology capital, something that does not affect the qualitative implications of the model but is necessary to obtain realistic amounts of FDI when undertaking the quantitative analysis below.

Multinationals. In both countries, there is a large number of firms, labelled multinationals because of their ability to potentially produce both at home and abroad. The mass of firms is constant and normalized to one.[21] In each country, there is a large number of locations where production can take place.[22] The measure of locations is,

[21] We do abstract from entry and exit considerations. One should think of it in the following way. If a domestic multinational wants to enter the domestic market, it has to buy the product or market by an existing multinational that has to exit. In that way, the mass of firms active stays constant. Please also note that we do not allow domestic firms to buy other firms' assets or product lines. This is an interesting future line of research.

[22] The derivation of the multinationals' production technology follows closely McGrattan and Prescott (2009); see also Kapicka (2012).

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without loss of generality, normalized to one. In each location, both domestic and foreign multinationals can set up a plant and operate. The production of a plant owned by a domestic multinational in a given location i depends on firm specific productivity z_t, labor services $l_t(i)$ and physical capital $k_t(i)$ and is given by a decreasing returns to scale technology $y_t(i) = e^{z_t}(k_t(i)^\theta l_t(i)^{1-\theta})^{1-\phi}$ with $0 < \phi < 1$. While physical capital and labor are both specific to each multinational and plant, technology capital M_t and productivity z_t is specific to each multinational only. The productivity of the foreign multinational is denoted by z_t^*. Technology capital and productivity therefore affect production in all locations, both domestic and foreign, in which the firm operates. A home multinational with M_t units of technology capital, k_{dt} units of domestic physical capital, and l_{dt} units of domestic labor services efficiently allocates physical capital and labor across all M_t domestic plants. Therefore, its total production in the home country is given by

$$y_{dt} = e^{z_t} M_t^\phi \left(k_{dt}^\theta l_{dt}^{1-\theta}\right)^{1-\phi}. \tag{2.14}$$

Technology capital can also be used to set up operations in a foreign location. Foreign owned multinationals accumulate domestic physical capital and hire domestic labor services and use their own technology capital. In contrast to domestic firms, the production of a foreign multinational depends on the countries' FDI openness. The degree of openness to FDI for both countries is given by a parameter τ that determines the total average factor productivity of a foreign multinational relative to a domestic multinational.[23] To illustrate this point, consider a multinational owned by the domestic consumer with given technology capital M_t and productivity z_t. It allocates efficiently its foreign physical capital k_{ft} and foreign labor services l_{ft} to generate total output abroad given by

$$y_{ft} = \tau e^{z_t} M_t^\phi \left(k_{ft}^\theta l_{ft}^{1-\theta}\right)^{1-\phi}. \tag{2.15}$$

Analogously, a foreign owned multinational with M_t^* units of technology capital and productivity z_t^*, k_{ft}^* units of home country's physical capital, and l_{ft}^* units of home country's labor services produces total output in the home country according to

$$y_{ft}^* = \tau e^{z_t^*} (M_t^*)^\phi \left((k_{ft}^*)^\theta (l_{ft}^*)^{1-\theta}\right)^{1-\phi}. \tag{2.16}$$

The domestic multinationals' total dividends are then given by the proceeds from their domestic and foreign operations, respectively, or

$$d_{Mt} = (p_{dt} y_{dt} - w_t l_{dt} - i_{dt}) + (p_{ft}^* y_{ft} - w_t^* l_{ft} - i_{ft}) - i_{Mt} \tag{2.17}$$

where the inverse demand functions p_{dt}, p_{ft}^* of domestic and foreign intermediate good producers defined in (2.13) are taken as given; i_{dt} and i_{ft} represent investment in domestic and foreign physical capital, respectively, and i_{Mt} represents the multinationals'

[23] Here, we impose symmetry across countries and assume that both countries have the same degree of openness.

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investment in technology capital. The respective capital stocks evolve according to

$$k_{jt+1} = (1-\delta)k_{jt} + \left[\frac{\chi_1 \left(\frac{i_{jt}}{k_{jt}}\right)^{1-\psi}}{1-\psi} + \chi_2 \right] k_{jt} \qquad j = d, f \tag{2.18}$$

$$M_{t+1} = (1-\delta_m)M_t + \left[\frac{\chi_1^m \left(\frac{i_{Mt}}{M_t}\right)^{1-\psi_m}}{1-\psi_m} + \chi_2^m \right] M_t \tag{2.19}$$

where δ_m is the depreciation rate of technology capital, ψ_m determines the sensitivity of the cost to investment in technology capital, and the parameters χ_1^m and χ_2^m are set by imposing steady state targets.[24] Note that the parameters for the adjustment costs in physical capital are identical across sectors. To complete the description of the multinationals problem, I assume that the log of domestic and foreign multinationals' productivity evolves according to a bivariate auto-regressive process

$$\begin{bmatrix} z_t \\ z_t^* \end{bmatrix} = \Lambda_z \begin{bmatrix} z_{t-1} \\ z_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t^z \\ \varepsilon_t^{z^*} \end{bmatrix} \tag{2.20}$$

where Λ_z is a 2×2 matrix and $[\varepsilon_t^z, \varepsilon_t^{z^*}]'$ is a vector of i.i.d. random variables with mean 0, standard deviation σ_z and correlation ρ_z. Multinationals resident in the home country solve

$$\max E \sum_t^\infty Q_t d_{Mt}$$

subject to (2.13),(2.17), (2.18), (2.19), M_0, k_{d0}, k_{f0} given, where Q_t again is the marginal utility of consumption of the domestic consumers (who are the owners). The problem of foreign owned multinationals is analogous.

2.3.2 Equilibrium

An equilibrium, for an exogenously given level of FDI openess τ, is a collection of price sequences p_{dt}, p_{dt}^*, p_{ft}, p_{ft}^*, P_t, P_t^*, Q_t, Q_t^* , $q(s^t, s_{t+1})$ $\forall s_{t+1} \in S$, exogenous shock processes z_t, z_t^*, a_t, a_t^* and quantities C_t, L_t, i_{Tt}, i_{dt}, i_{ft}, i_{Mt}, l_{Tt}, l_{dt}, l_{ft}, d_t, x_{dt}, x_{ft}, y_{dt}, y_{ft}, X_t, Y_{It}, Y_t, $B(s^t, s_{t+1})$, C_t^*, L_t^*, i_{Tt}^*, i_{dt}^*, i_{ft}^*, i_{Mt}^*, l_{Tt}^*, l_{dt}^*, l_{ft}^*, d_t^*, x_{dt}^*, x_{ft}^*, y_{dt}^*, y_{ft}^*, X_t^*, Y_{It}^*, Y_t^*, $B^*(s^t, s_{t+1})$ $\forall s_{t+1} \in S$ such that:

1. Given prices and shocks, consumers and firms solve their respective problems.

2. Labor markets clear, i.e.

$$L_t = l_{dt} + l_{ft}^* + l_{Tt} \qquad \text{for all } t.$$

$$L_t^* = l_{dt}^* + l_{ft} + l_{Tt}^* \qquad \text{for all } t.$$

[24] I set the parameters such that the depreciation rate in steady state is equal to δ_m and the derivative of technology capital with respect to investment is equal to one.

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3. Intermediate goods markets clear, i.e.

$$X_t = Y_{It} \qquad X_t^* = Y_{It}^* \qquad \text{for all } t.$$
$$x_{dt} = y_{dt} \qquad x_{ft} = y_{ft}^* \qquad x_{dt}^* = y_{dt}^* \qquad x_{ft}^* = y_{ft} \qquad \text{for all } t.$$

4. Under complete financial markets bond markets clear, i.e.

$$B(s^t, s_{t+1}) + B^*(s^t, s_{t+1}) = 0 \qquad \text{for all } t, s_{t+1} \in S.$$

5. The tradable goods market clears, i.e.

$$C_t + C_t^* + i_{Tt} + i_{Tt}^* + i_{dt} + i_{ft} + i_{dt}^* + i_{ft}^* + i_{Mt} + i_{Mt}^* = Y_t + Y_t^* \qquad \text{for all } t.$$

2.3.3 National accounts and measured returns

Because in national accounts investment in technology capital is expensed,measured gross domestic product in the home country is given by[25]

$$GDP_t = Y_t - i_{Mt}. \tag{2.21}$$

This means that GDP differs from actual value added Y_t whenever investment in technology capital is different from zero. This also implies that the dynamic properties of GDP - in particular cross country correlations - depend both on output and investment in technology capital.

Gross FDI positions are given by

$$FDIA_t = k_{ft} \qquad FDIL_t = k_{ft}^*$$

and total bilateral FDI linkages are computed in line with the empirical estimates[26]

$$FDI/GDP = \frac{2(FDIA_t + FDIL_t)}{4(GDP_t + GDP_t^*)}.$$

In terms of measurement, other key variables are the returns on FDI. Returns reported in balance of payment statistics e.g. by the BEA do not coincide with the *actual* returns

[25]The equation follows by adding up aggregate labor income $w_t L_t$, firms' dividends and depreciation of physical (or tangible) capital. The crucial assumption is that technology capital is intangible and therefore not taking into account when computing aggregate income. See also McGrattan and Prescott (2010).

[26]Note that quarterly GDP is annualized because in national accounts, quarterly gross domestic product is reported at annualized levels.

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multinationals receive from foreign direct investment.[27] To see this in the present setup, consider the actual return domestic multinationals receive from their subsidiaries abroad

$$r_{ft} = \theta\eta(1-\phi)\frac{p^*_{ft}y_{ft}}{k_{ft}} - \delta.$$

In the data, measured returns of foreign subsidiaries from the abroad are computed as FDI income (dividends plus reinvested earnings) divided by the tangible capital stock owned by the multinationals. In the notation of my model, measured returns for the domestic multinational from its subsidiaries abroad are given by

$$r_{FDI,t} = \frac{p^*_{ft}y_{ft} - w^*_t l_{ft} - \delta k_{ft}}{k_{ft}} = r_{ft} + (1-(1-\phi)\eta)\frac{p^*_{ft}y_{ft}}{k_{ft}}. \tag{2.22}$$

As the returns on technology capital are not taken into account, measured returns differ from the actual returns by the second term in the above expression. In order to calibrate of the multinational-specific shock, I will match the volatility of measured returns pubilshed by the BEA, as outlined in more detail in the next sub-section.

2.3.4 Calibration

The equilibrium described above does not admit an analytical solution. I therefore derive a numerical solution using standard linearization techniques. For this purpose, I need to assign numerical values to the various parameters. Table 2.4 shows the parameters used in the calibration for the three model specifications considered: (1) the model with complete financial markets and country-specific productivity shocks only, (2) the model with complete financial markets with both country-specific productivity shocks and mutlinational-specific shocks to the efficiency of technology capital, and (3) financial autarky with both shocks.

The discount factor is set to $\beta = 0.99$ implying an average interest rate of 4 percent. The share of intermediate inputs in final tradable production ν is set to 0.5, following Alvarez and Lucas (2007).[28] For the demand elasticity of substitution in the intermediate sector I follow Atkeson and Kehoe (2005) and set η to 0.9, implying mark-up of 11 percent and an elasticity of substitution of 10. For the depreciation of physical capital, I choose a standard value and set δ equal to 0.025. Regarding the depreciation of technology capital,

[27] This was first pointed out by McGrattan and Prescott (2010).

[28] Even though in their model the only input in production is labor, the model is calibrated in a way that is perfectly consistent to my setup, as they compute the share of effective labor which includes capital.

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Table 2.4: Parameter values

Description	Symbol	Model: Complete Markets	Complete markets, both shocks	Financial autarky, both shocks
Preferences				
Discount factor	β	0.99		
Total time endowment	$\bar{L}$	3		
Weight of leisure	α	1.602	1.602	0.955
Technology				
Income share of labor in production	θ	0.31		
Share of intermediate goods in tradables	ν	0.50		
Income share of technology capital	ϕ	0.21		
Elasticity of demand interm. good sector	$\frac{1}{1-\eta}$	10		
Degree of FDI openness	τ	0.22	0.22	0.22
Deprectiation and adjustment costs				
Depreciation physical capital	δ	0.025		
Depreciation technology capital	δ_m	0.0375		
Adjustment cost physical capital	ψ	0.097	0.099	0.138
Adjustment cost technology capital	ψ_m	0.090	0.095	0.122
Std. dev. prod. shock	σ_a	0.008	0.004	0.008
Std. dev. techn. capital efficiency shock	σ_z	0.000	0.015	0.011
Cross-country correlation prod. shock	$\rho_a = \rho_z$	0.600	0.600	0.400
Autoregessive coefficients	$\Lambda_a = \Lambda_z$	$\begin{bmatrix} 0.95 & 0 \\ 0 & 0.95 \end{bmatrix}$		

I use $\delta_m = 0.0375$, implying an annual depreciation rate of 15 percent, that is, the BEA estimate for depreciation of R&D capital.[29]

As outlined in the following paragraphs, the remaining parameters are chosen in order to match key moments of the data. This includes also the parameters for the stochastic processes because - even in the model with country-specific productivity shocks only (a_t, a_t^*) - there is not a one-to-one mapping between the stochastic processes and the

[29]See Eisfeldt and Papanikolaou (2013). Also McGrattan and Prescott (2010) or Kapicka (2012) assume that technology capital depreciates faster than physical capital. For the main results of this paper, this assumption is not crucial. The quantitative implications are affected but not sensitive for reasonable perturbations of this parameter (between 0.01 and 0.15).

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Table 2.5: Targeted data moments and model fit

	Data	Model		
		Complete Markets	Complete markets, both shocks	Financial autarky, both shocks
Long-run averages				
Labor income share	64	64	64	64
Investment in technology capital over GDP	8	8.0	8.0	8.0
FDI/GDP	1.5	1.5	1.5	1.5
Second moments				
Std. dev. of GDP	1.3	1.3	1.3	1.3
Std. dev. of investment in physical capital relative to GDP	3	3.0	3.0	3.0
Std. dev. of investment in technology capital relative to GDP	3	3.0	3.0	3.0
Cross-country GDP correlation	0.4	0.4	0.4	0.4
Std. dev. FDI returns	1.6	0.9	1.6	1.6

Notes: This table presents the target moments used for the calibration. I compare the moments in the data to the averages of the models' stationary distributions obtained by simulating 150000 time periods and dropping the first 50000. The labor income share and returns on FDI are computed from US data, published by the BEA. The remaining data moments are the median values across time and countries for the OECD sample, see table 2.1. For a detailed description of the data see appendix B.1. Column two shows the data moments. Column three refers to the model with complete financial markets and country-specific productivity shocks in the tradable sector only. Column four shows the implied moments from the model with complete financial markets and both country-specific productivity shocks in the tradable sector and multinational specific shocks to the efficiency of technology capital. Column five shows the moments for the model under financial autarky and both shocks.

Solow residual obtained from the data because GDP is mismeasured. Table 2.5 reports the data targets and the model fit.

Utility and production. The weight on leisure in the utility function α is set so that households, on average, work one third of the available time. By normalizing the total time endowment to $\bar{L} = 3$, this implies a long-run target for employment equal to one. For the share of technology capital in multinationals' production (ϕ), I follow McGrattan and Prescott (2009) and match average investment in technology capital over GDP equal to eight percent. The share of capital in production (θ) is then set so that in steady state the labor share is 64 percent. The degree of FDI openness (τ) is set so that the bilateral FDI position in the model - measured as in the data according to equation (2.4) - is equal

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to 1.7 percent, that is, the median value of bilateral FDI linkages in the data, shown in table 2.1 row five, column seven.

Adjustment costs. Adjustment costs for both physical capital (ψ) and technology capital (ψ_m), respectively, are set so that the hp-filtered investment series of both types of capital are three times as volatile as hp-filtered GDP. For physical capital, this is a standard value and consistent with OECD data. For technology capital, the number requires some discussion. Ouyang (2011) reports that the growth rate of R&D expenditures are 1.6 times as volatile as GDP growth rates; on the other hand, Eisfeldt and Papanikolaou (2013) find that the volatility of the investment rate in organizational capital is 1.5 percent (annual).[30] Targeting the latter value would imply in this model that investment in technolgy capital was six times as volatile as GDP. Because I adopt the broader definition of technology capital (marketing expenditures plus organizational capital plus R&D) and R&D is the largest part in technology capital,[31] I choose a value in between and assume that investment in technology capital behaves similar to investment in physical capital in terms of volatility, in line with the findings in Wälde and Woitek (2004) for R&D investment and G7 data. It is worth noticing that the selected adjustment costs on investment in physical capital are quiet low compared to standard business cycle models; the presence of technology capital makes investment in physical capital less volatile.

Exogenous shocks. For the stochastic processes I assume that the transition matrices are the same $\Lambda_a = \Lambda_z$, with a value of 0.95 on the diagonals and zero on the off-diagonals. Multinational-specific productivity shocks are perfectly correlated within countries (all domestic multinationals have the same productivity) and not correlated across countries (foreign multinationals' productivity follows a statistically independent stochastic process). The volatility of productivity in the tradable sectore (σ_a) and its cross-country correlation (ρ_a) are set so that measured GDP in the model matches the standard deviation of GDP in the data (1.3 percent) and the median value of the hp-filtered cross-country GDP correlation in the data (0.5, see table 2.1), respectively. For the model specifications with multinational-specific shocks (z_t, z_t^*), I choose the volatility σ_z such that the volatility of measured FDI returns as defined in equation (2.22) matches the reported volatility in the data, equal to 1.6 percent annualized.[32]

[30] They define the investment rate as aggregate investment divided by the existing capital stock, see Eisfeldt and Papanikolaou (2013) table II.

[31] See McGrattan and Prescott (2010).

[32] Note that the estimate in the data comes from the BEA and is the average over US inward and outward FDI income (see data appendix for a description of the data).

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2.3.5 Impulse responses

This subsection presents the dynamic responses to the two types of shocks in separation. This serves two purposes. First, I show in what respects multinational-specific shocks differ from country-specific productivity shocks, in terms of model dynamics. Second, this allows me to illustrate how the transmission of these two shocks changes as countries open up to FDI. I therefore present both the impulse responses for country pairs with relatively little FDI linkages (i.e. a low τ) and countries with large FDI linkages (i.e. high τ).

Productivity shocks

Figure 2.1 shows the impulse responses to a positive productivity shock in the home country's tradable sector for value added, measured GDP, and investments in physical and technology capital. Blue lines refer to the domestic country, the red lines to the foreign country. Solid lines refer to a country pair that is relatively closed to FDI, dashed lines refer to a relatively open country pair in terms of FDI.[33] Three observations emerge.

First, both actual value added (panel a) and GDP (panel b) increase in the home country while both show relatively little response in the foreign country. At the same time, investment in physical capital (panel c) and investment in technology capital (panel d) increase in the home country and fall in the foreign country; notably, the on-impact increase of investments in the home country exceeds the fall in the foreign country by a factor four.

Second, technology capital behaves differently to a domestic productivity shock when countries are relatively more open to FDI (dashed lines); while the on-impact increase in technology capital investment in the home country decreases from four to two and a half percent, the response of foreign investment in technology capital switches sign and actually turns positive. Hence, when countries are relatively more open to FDI, a positive productivity shock in the home country benefits both domestic and foreign multinationals, whose dividends increase. Because the shocks are persistent, the expected returns on technology capital increase. Within a firm, in turn, all returns are equalized, therefore the returns both on foreign and domestic investment (from the firms' perspective) increase. As a consequence, foreign multinationals invest not only more in technology capital, but also more in physical capital both at home and abroad. On the other hand, because part of the additional returns of the increased domestic productivity benefits foreign firms,

[33]For space considerations, the shock process itself is not shown. I consider a one-standard deviation shock in period 1; all plotted responses are normalized by the standard deviation of the shock.

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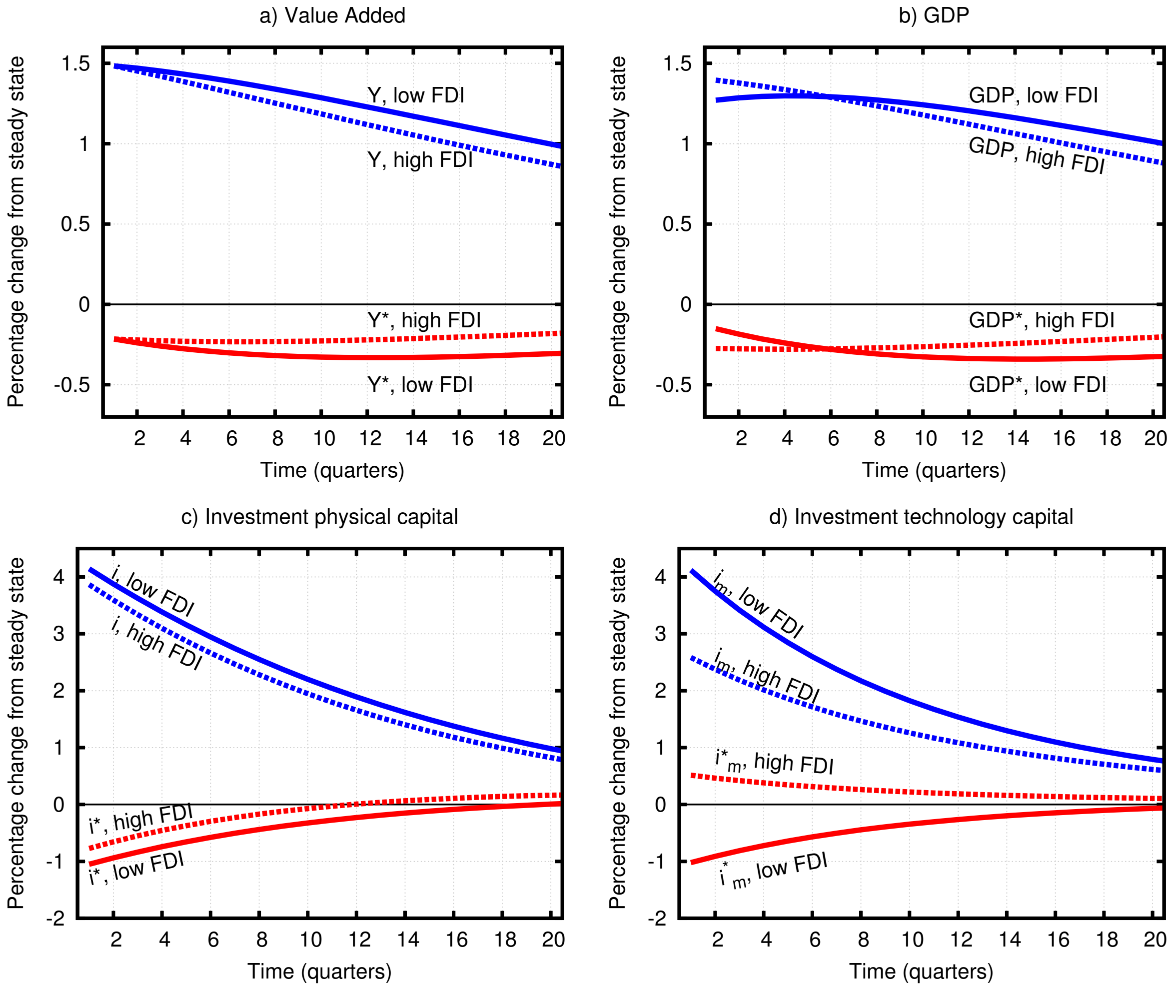
Figure 2.1: Impulse responses to a productivity shock: low versus high bilateral FDI linkages



Notes: The figure shows impulse responses to a one-standard deviation positive productivity shock of one standard deviation in the home country (ε_t^a). All responses are in percentage deviations from the steady state, normalized by the standard deviation of the shock. Solid lines refer to the relatively closed country pair (low τ), dashed lines refer to the relatively open country pair (high τ). Blue lines refer to the home country, red lines to the foreign country. Panel a) plots the impulse responses for value added (Y_t and Y_t^*) as defined in equation (2.8), panel b) for measured GDP (defined as $Y_t - i_{mt}$), panel c) for investment in physical capital ($i_t = i_{Tt} + i_{dt} + i_{ft}^*$ and $i_t^* = i_{Tt}^* + i_{dt}^* + i_{ft}$), and panel d) for investment in technology capital (i_{Mt} and i_{Mt}^*).

domestic firms increase their investments by a smaller amount than it would be the case when countries are relatively closed to FDI. The previous discussion is reflected by the responses of countries' aggregate investment in physical capital shown by the dashed lines in panel c). The responses shift move closer together, meaning that domestic and foreign investment become less negatively correlated.

Third, actual value added and measured GDP - shown in panels a) and b) - do qualitatively not respond differently when varying the countries' openness to FDI (compare the solid versus dashed lines). The only notable difference is the on-impact response in

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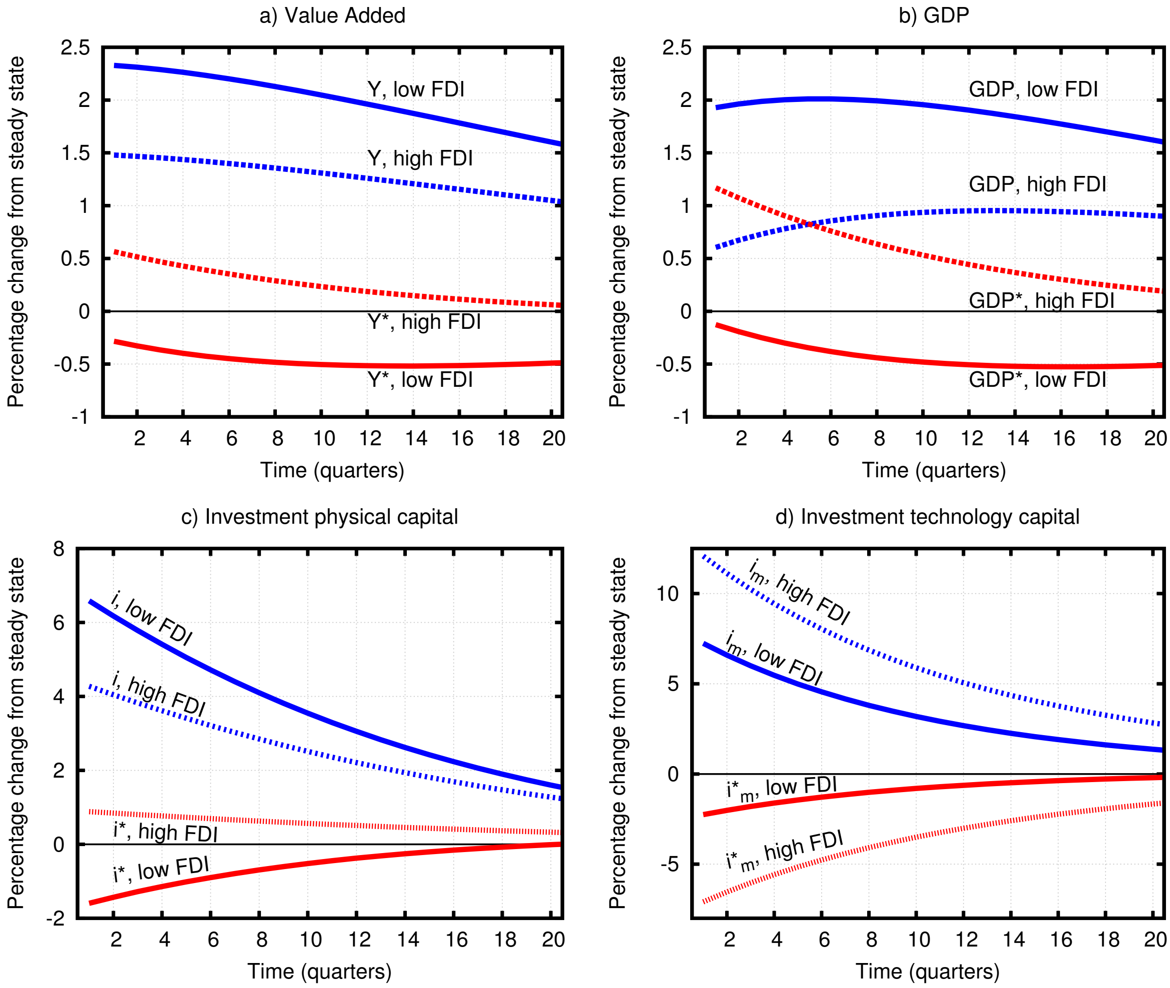
Figure 2.2: Impulse responses to a positive shock to multinational activity: low versus high bilateral FDI linkages



Notes: The figure shows impulse responses to a one-standard deviation positive shock to the efficiency of domestic multinationals (ε_t^z). All responses are in percentage deviations from the steady state and normalized by the standard deviation of the shock. Solid lines refer to the relatively closed country pair (low τ), the dashed lines refer to the relatively open country pair (high τ). Blue lines refer to the home country, red lines to the foreign country. Panel a) plots the impulse responses for value added (Y_t and Y_t^*) as defined in equation (2.8), panel b) for measured GDP (defined as $Y_t - i_{mt}$), panel c) for investment in physical capital ($i_t = i_{Tt} + i_{dt} + i^*_{ft}$ and $i_t^* = i^*_{Tt} + i^*_{dt} + i_{ft}$), and panel d) for investment in technology capital (i_{Mt} and i^*_{Mt}).

measured GDP: because investment in technology capital responds less positive in the home country compared to the closed case, GDP increases by more on impact (and the reverse for the foreign country). The transition dynamics however are largely unaffected, suggesting that GDP correlations become somewhat more negative with increasing FDI integration.

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Shocks to multinational activity

Figure 2.2 shows the impulse responses to a positive shock to domestically owned multinationals' productivity (ε_t^z) for value added, measured GDP, and investments in physical and technology capital, respectively. Blue lines refer to the domestic country, red lines to the foreign country. The solid lines refer to a country pair that is relatively closed to FDI, dashed lines refer to a relatively open country pair in terms of FDI.[34]

For relatively closed countries (solid lines) multinational-specific shocks work very much like standard productivity shocks, moving all quantities in a similar fashion as described in the previous sub-section. The reason is that in this case the share of foreign firms is too small to affect aggregate quantities.

On the other hand, the picture changes when countries are relatively open to FDI (dashed lines). First, consider the responses of investment in technology capital shown in panel d). The dashed blue lines lies above the solid blue line, meaning that domestic multinationals increase their investment in technology capital by more than in the closed economy case. The reason is that domestic multinationals gain from investments in technology capital because of the relatively higher returns from abroad. Similarly, the red dashed line lies below the red solid line meaning that foreign multinationals investments in technology capital decrease by more than compared to the case with low FDI linkages.

This, in turn, affects the profitability of investment in physical capital, so it is rational for the multinationals to invest more at home and abroad. This is reflected by the increase in aggregate investment both at home and abroad, as shown by the dashed lines in panel c). Note that this also implies that the increase in investment in physical capital in the home country is below the solid blue line in panel c) because part of the resources get redirected to the foreign country. As a consequence, international investment becomes more synchronized when countries are relatively open to FDI. Hence, multinational-specific shocks exacerbate the effect of technology on the investment comovement.

Finally, consider the responses of value added in panel a) and GDP in panel b) in the high FDI linkage scenario (dashed lines). The first notable observation is that value added in both countries increase in response to the shock. The reason is that part of the resources are shifted to the foreign country in form of additional investment of home owned multinationals leading to more output abroad. This is also reflected by the less positive response of value added in the home country (the dashed blue line lies below the solid blue line). Measured GDP - as shown by the dashed lines in panel b) - reflects not only

[34]For space considerations, the shock process itself is not shown. I consider a one-standard deviation shock in period 1; all plotted responses are normalized by the standard deviation of the shock.

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Table 2.6: Business cycle statistics

	GDP	Cons.	Investm.	Employm.	Net Exports
Volatilities		relative to GDP volatility			
Complete markets, prod. shock only	$1.3^{\dagger}$	0.47	$3.0^{\dagger}$	0.53	0.29
Complete markets, both shocks	$1.3^{\dagger}$	0.48	$3.0^{\dagger}$	0.53	0.30
Financial autarky, both shocks	$1.3^{\dagger}$	0.76	$3.0^{\dagger}$	0.26	0.11
Correlation with GDP					
Complete markets, prod. shock only		0.83	0.96	0.93	0.41
Complete markets, both shocks		0.83	0.96	0.92	0.40
Financial autarky, both shocks		1.00	0.99	0.97	-0.90
International Correlation					
Complete markets, prod. shock only	$0.4^{\dagger}$	1.00	0.19	-0.24	
Complete markets, both shocks	$0.4^{\dagger}$	1.00	0.18	-0.27	
Financial autarky, both shocks	$0.4^{\dagger}$	0.42	0.42	0.41	

Notes: † denotes statistics matched in the calibration.

the movements of value added but also the responses in investment in technology capital. Because domestic investment in technology capital responds relatively more when FDI linkages are high, the impulse response of domestic GDP shifts towards South-East such that it even crosses the response of foreign GDP. That means that - on impact - measured GDP abroad increases by more than domestic GDP. In the subsequent transition then, domestic GDP rises, while foreign GDP falls. This suggests that measured GDP might get even more negatively correlated when FDI openness increase.

2.4 Quantitative Results

2.4.1 Business cycle properties

In this section, I use the model to assess the business cycle implications of technology capital and shocks to multinational activity. For this purpose, I compare the long-run business cycle statistics for the three model specifications. Table 2.6 reports the results of the quantitative exercise. The rows labelled "Complete Markets, prod. shock only" report business cycle statistics for the model with complete financial markets and productivity shocks only; the rows labelled "Complete markets, both shocks" refers to the model with complete markets and both productivity shocks and shocks to the efficiency of technology capital; the rows labelled "financial autarky, both shocks" refer to the financial autarky

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model with both shocks.

There are three notable results. First, all model versions generates business cycle statistics similar to those of standard international business cycle models which includes the well known short-comings. The complete market models imply perfect cross-country correlation of consumption, negative cross-country employment correlations, and pro-cyclical net exports. Interestingly, the complete market models generate volatilities of net exports in line with the data, while in the model with financial autarky volatility of net exports is too low compared to the data.

Second, and quiet interestingly, the cross-country correlation of consumption under financial autarky is equal to the correlation of GDP. This means the model can to some extend address the so called "quantity anomaly", that is, the fact that in the standard business cycle model consumption correlations are more positive than GDP correlations while in the data the opposite is true. One reason for this finding is the measurement issue for GDP. Recall that GDP is value added minus investment in technology capital. We will see below that when countries are relatively closed, measured GDP over-estimates the cross-country correlation of aggregate activity. This means that cross-country correlation of actual output is lower. In addition, when countries are relatively closed to FDI, cross-country correlations of consumption is closer to actual value added. The other reason is the presence of multinational-specific shocks that are uncorrelated across countries. Absent other international assets than FDI, households cannot insure this risk and consumption correlations are lower than without these shocks. I conjecture that for the same reasons the financial autarky model implies a cross-country correlation of employment in line with the data.

Third, the introduction of multinational-specific shocks leaves the business cycle moments mostly unaffected while it helps to match the volatility of measured returns. This confirms to some extent the validity of my calibration strategy where the volatility of the multinational-specific shocks was chosen in order to match the standard deviation of measured returns on FDI.

To summarize, the results in this section show that introducing technology capital and multinational-specific shocks into a standard international business cycle model generates plausible business cycle statistics, and helps to explain some features of the data the standard model has difficulties with.

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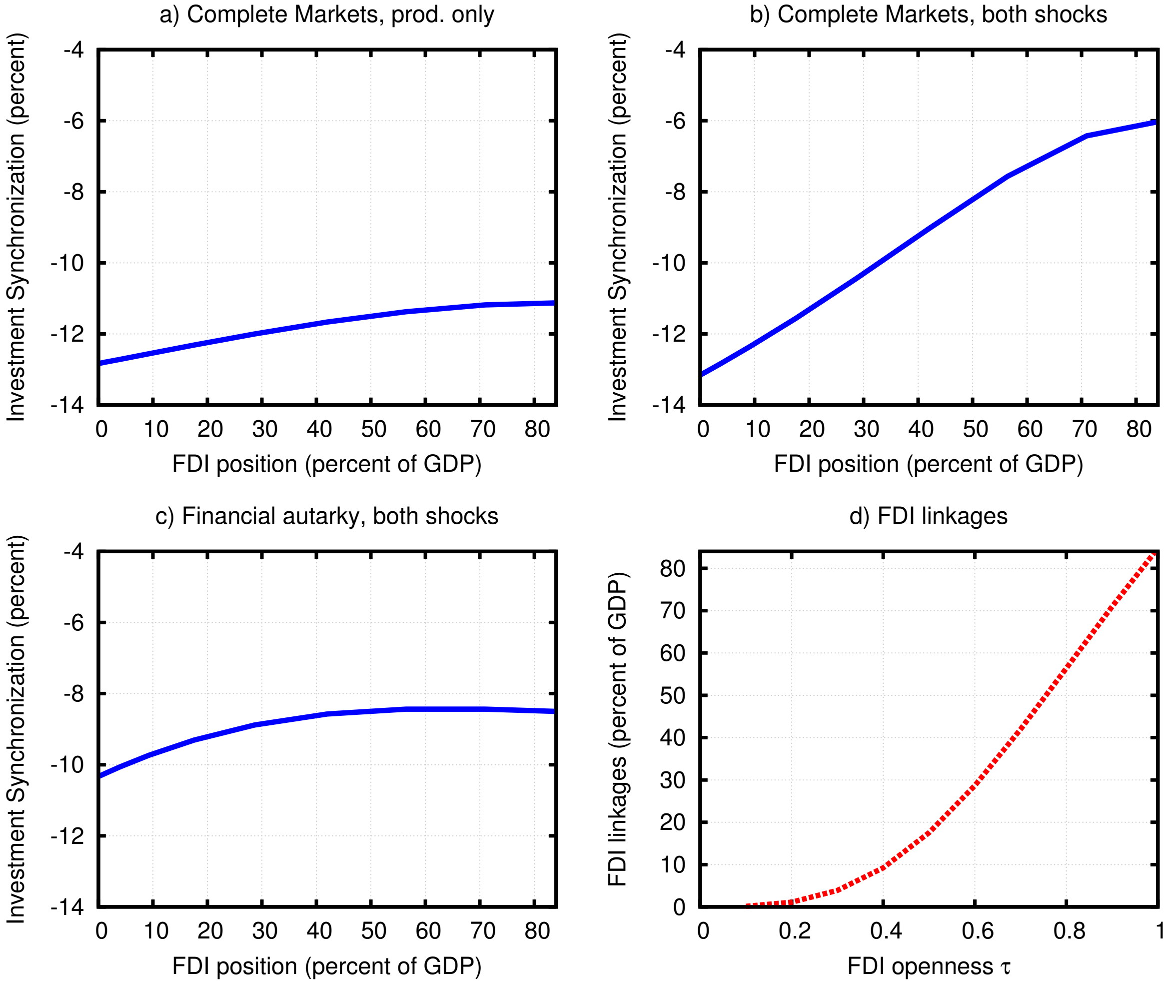
Figure 2.3: FDI openness and investment synchronization



2.4.2 FDI integration and business cycle synchronization

This sections connects the quantitative results of the model with the empirical results in the first part of the paper. I start by varying the FDI openness parameter τ and discuss how FDI openness affects co-movement of investment and GDP, respectively. Second, using artificial data generated by the model, I run the same regressions as in the data and compare the obtained regression coefficients. This is a simple test whether the causal relation in the model is consistent with the data. In addition, the model allows me to distinguish between measured GDP as reported in national accounts and actual value added and I will show how their co-movement patterns differ.

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FDI openness and investment synchronization

For each model version as described above (complete markets with productivity shocks only, complete markets with both shocks, and financial autarky with both shocks), I vary the degree of FDI openness from very low integration ($\tau = 0.05$) to complete FDI openness ($\tau = 1$).[35] For each value of the financial integration parameter I report the average synchronization of investment as defined in equation (2.3). Figure 2.3 shows the results of this exercise. In all model versions, the slope of the line is always positive; a higher degree of FDI openness leads to more correlated investment cycles. This result is consistent with my regression estimates in table 2.2.

There are three main conclusions from the comparison of the three model versions. First, there is a clear ranking in terms of the slope of the increase in investment synchronization: the smallest in the model with complete markets and productivity shocks only (panel a)) and the strongest in complete markets and both productivity and technology capital shocks (panel b)). This suggest that both technology capital and multinational-specific shocks are quantitatively important for understanding investment co-movement.

Second, the slope of the increase in investment synchronization under financial autarky (panel c)) lies in between the two models with complete markets.[36] When households cannot trade any financial assets (recall that FDI is assumed to be the only asset and is undertaken by firms) and both shocks are active, there are two counter-acting forces: on the one hand, in response to country-specific productivity shocks, investment tends to flow to the more productive country and this weakens investment co-movement; on the other hand, when openness increases, firm-specific shocks to the efficiency of technology capital become more important and returns on investment within firm become more correlated. As described in section 2.3.5 this strengthens investment co-movement. As is evident in panel c), the latter force is dominating for most of the range of FDI openness τ.

Third, in all model versions, the increase in investment synchronization is fastest for middle ranges of FDI openness τ between 0.2 and 0.8; the reason behind this is the fact that, for these values of τ, the gains from FDI are biggest and FDI positions increase the fastest, as shown in panel d).

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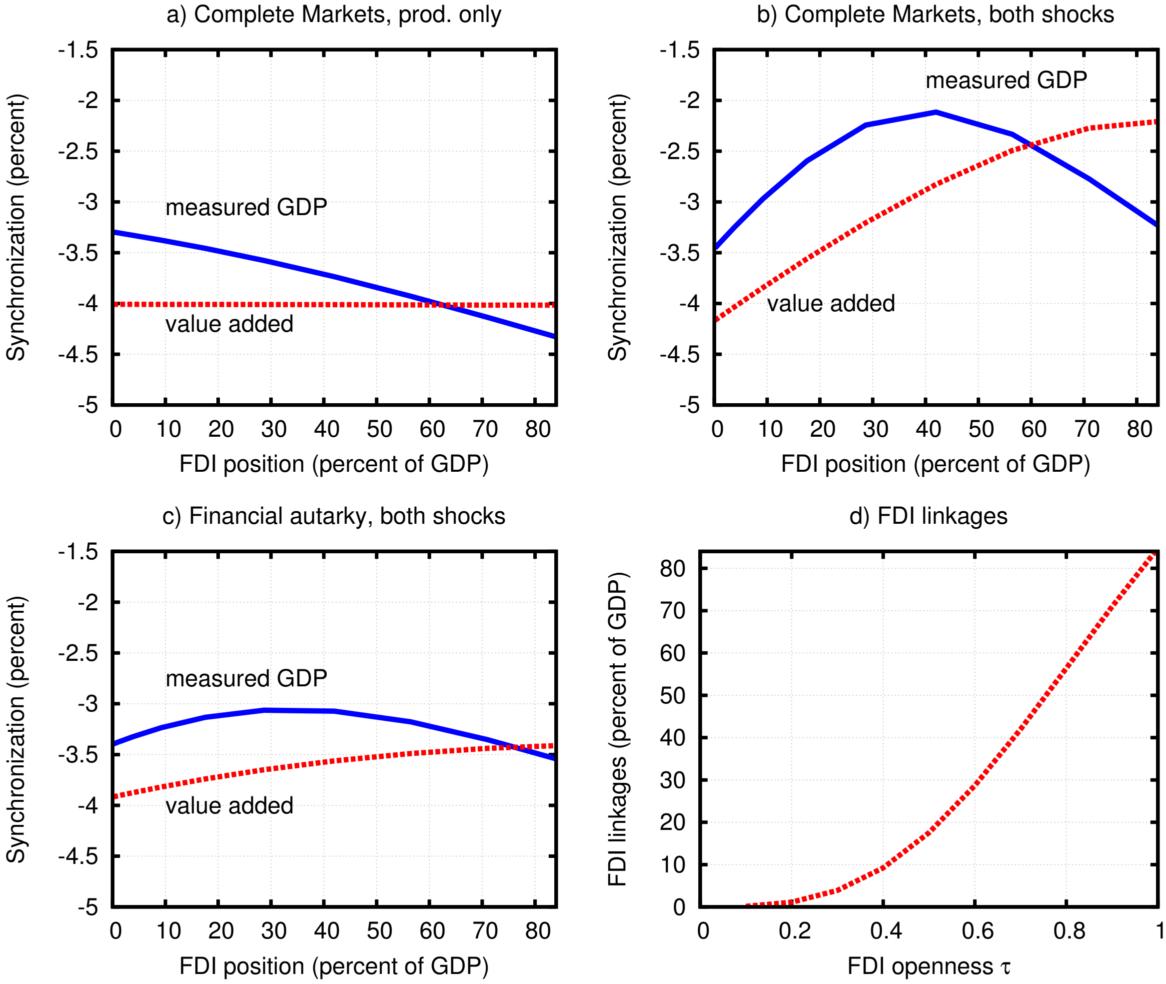
Figure 2.4: FDI openness and GDP synchronization



FDI openness and GDP synchronization

Let us now turn to the synchronization patterns of GDP as measured in national accounts (equation (2.21)) and actual value added (equation (2.8)). Again, for each model version (complete markets with productivity shocks only, complete markets with both shocks, and financial autarky with both shocks), I vary the degree of FDI openness from very low integration ($\tau = 0.05$) to complete FDI openness ($\tau = 1$).[37] For each value of the financial integration parameter I report the average synchronization of GDP and value added as defined in equation (2.2). The results are shown in figure 2.4.

[35] I take a value for τ that is slightly bigger than zero for numerical reasons; a value of $\tau = 0.05$ implies a bilateral FDI to GDP ratio of 0.015 percent, i.e. the 2.5 percent quantile in the data.

[36] When considering financial autarky with productivity shocks only, investment synchronization responds slighlty negative in response to variation in τ (not shown), see appendix.

[37] See footnote 35.

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In all model versions, synchronization patterns of measured GDP (blue solid lines) and value added (dashed red lines) behave quiet differently: while GDP shows a non-linear pattern in response to variations to FDI openness τ, synchronization of value added is constant (panel a)) or monotone increasing (panels b) and c)). This means that - due to the mismeasurement in GDP - actual cross country co-movement is overestimated for relatively closed country-pairs and underestimated when countries are relatively open to FDI.

We can gain some intuition by answering the following two questions. First, under complete markets with productivity shocks only (panel a)), why is GDP synchronization falling when FDI opennes increases? When FDI openness increases, a positive productivity shock in one country benefits multinationals from both countries due to their increased operations abroad. On impact, because expected returns increase, multinationals from both countries increase their investment in technology capital. The more the countries are open, the more similar in terms of size is this increase in investment in technology capital. Domestic investment in technology capital increases by relatively less and foreign investment in technology capital increases by relatively more when comparing a relatively open country pair to a relatively closed country pair. Everything else equal, by the definition of GDP (value added minus investment in technology capital), measured GDP at home is then relatively higher and GDP abroad is relatively lower; GDP is more negatively correlated.

Second, when looking at panels b) and c), why is GDP synchronization first increasing (for values of τ below 0.6) and then falling again (for values of τ above 0.6)? Consider first the complete market case as shown in panel b). In this model version, both country-specific and multinational-specific shocks are active. In the range of τ between 0.2 and 0.6 the returns of increasing FDI is biggest as discussed above and also shown in panel d). Therefore in this range, the country-specific and multinational-specific shock work in the same way and measured GDP resembles the synchronization pattern of value added (while the correlation of investment in technology capital is also increasing, see figure 2.1). Recall that following a positive shock to the domestic multinational, value added of both countries increases because the multinational invests in both countries more. At τ equal to approximately 0.6, however, there is a dipping point. Multinational-specific shocks dominate and investments in technology capital become negatively correlated, even exceeding the increase in correlation of value added. By the definition of GDP (value added minus investment in technology capital), GDP correlation therefore decreases.

Finally, under financial autarky, the pattern of GDP and value added synchronization resembles the pattern under complete markets. However, the lines in panel c are flatter than the ones in panel b). Incomplete markets dampen the amplitude of the synchroniza-

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Table 2.7: Bilateral FDI linkages and synchronization: data vs. model

	Data (G7)	Model		
		Complete Markets	Complete markets, both shocks	Financial autarky, both shocks
Coefficient on investment synchronization	1.716 (2.78)	0.308	1.312	0.503
Coefficient on GDP synchronization	0.346 (1.41)	-0.136	0.253	0.081
Coefficient on synchron. of value added	n.a. n.a.	0.006	0.395	0.122

Notes: The table reports the estimated coefficients in the three model versions. For convenience, column one reports the estimated coefficients obtained from the data, taken from column (3) in tables 2.2 and 2.3, respectively. For the empirical estimates, t-values are reported in parenthesis below. The last three columns report the estimated coefficient of the three model versions. In the first row, the dependent variable is investment synchronization, in the second row GDP synchronization, and in the third row it is actual value added (Y). The right hand side variable in all regressions is log of the sum of bilateral FDI positions divided by the sum of the countries' GDP.

tion of GDP and value added by the same amount. This is because under this secenario, the comparative static exercise is not really 'ceteris paribus'. Besides the effects of technology flows associated with more FDI as described under complete markets, varying the parameter τ picks up another force: the possibility of countries to shift resources across country borders, hence completing the markets in terms of risk sharing. In this sense for each value of τ the agents face a different international market structure and the effect of FDI on investment and output co-movement is downward biased.

Regression coefficients in the model and data

In this section, I run similar regressions on model data to the ones in the empirical part of the paper. In particular, I simulate the model for ten country pairs, varying the FDI openness parameter (τ) smoothly from zero to one. For each country pair I simulate the model for 48 quarters (12 years as in the data) and construct the same measure of Investment and GDP synchronization and for bilateral FDI linkages as used in the data analysis. I then convert the quarterly data to the yearly frequency by averaging across quarters. Finally, I regress the obtained synchronization measures on the log of bilateral FDI linkages. Table 2.7 reports the results for the three model versions considered. Note that, in the last row of the table, I report the synchronization measure for actual

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value added; in the model value added differs from measured GDP because investment in technology capital is expensed in national accounts. In the data, I do not have an equivalent measure available, for this reason I put "n.a." in the respective column. For comparison in table 2.7 I also report the coefficients on the same regression using actual data, repeating the estimates in column (3) in tables 2.2 and 2.3, respectively.

Three results emerge. First, I find that overall a higher degree of FDI integration leads to higher level of investment synchronization, as reflected in an integration coefficient between 0.186 and 0.831. Therefore the model explains up to 20 percent of the estimated coefficient in the data (4.106).

Second, the the sign of relation between FDI integration and GDP synchronization is ambiguous; the estimated coefficients in the model range from -0.092 to 0.159 where the latter value is very close to the coefficient obtained from actual data (0.129). This suggests that the relation between financial integration and output co-movement implied by our model is statistically close to the one we estimate in the data. In terms of magnitude, the estimated coefficients for GDP synchronization are significantly smaller than the coefficients obtained for investment synchronization, as observed in the data.

Third, as reported in the last row of table 2.7, actual co-movement of aggregate activity is larger than suggested by the estimates for GDP synchronization: the regression coefficients on actual value added range from -0.001 to 0.261, always lying above the estimates for GDP synchronization. This means that focussing on measured GDP as an indicator for co-movement underestimates the actual business cycle co-movement. To get a sense for the magnitudes, consider the estimated coefficients for the model version with complete markets and both shocks, reported in column two. The coefficient for GDP synchronization is 0.159 while the one for actual value added is 0.261. Hence, when focussing on GDP, a doubling of FDI linkages would lead to an increase in GDP synchronization of 0.159 percentage points. In contrast, for value added a doubling of FDI linkages leads to an increase in synchronization of 0.261 percentage points, a value that is 65 percent higher than the one for GDP synchronization. Therefore the model gives a theoretical rationale for a substantial bias due to measurement error that emerges because national accounts do not include intangible capital.

2.5 Model implications for risk sharing and the 'quantity puzzle'.

In this section I discuss the model implications for two important issues in international macroeconomics. First, how does financial integration affect consumption risk sharing

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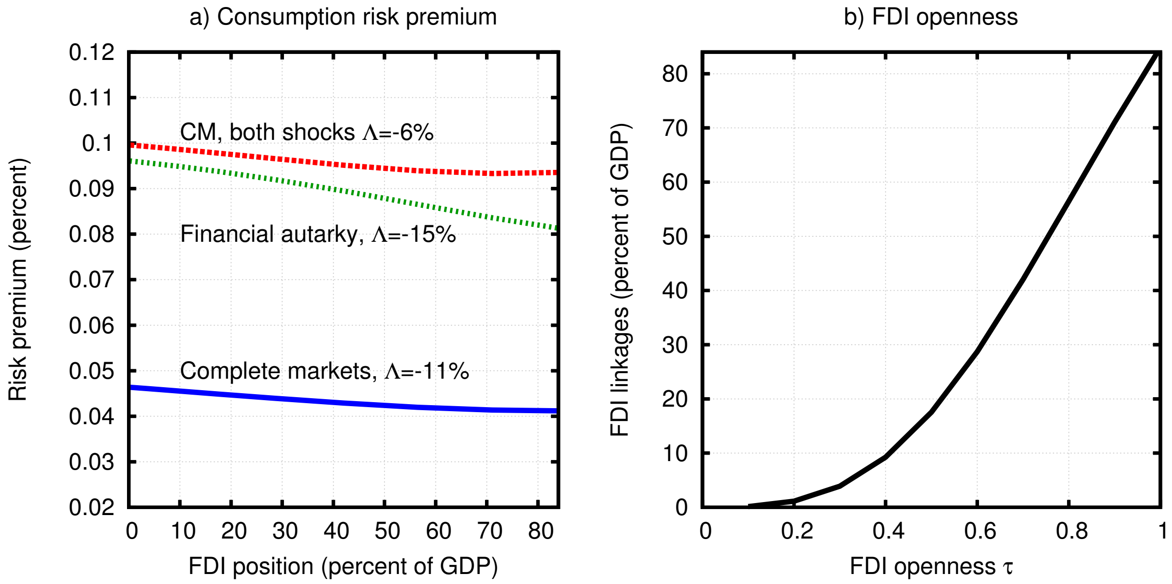
Figure 2.5: Consumption risk premium as function of FDI linkages



possibilities of countries? It is possible to say something about the risk sharing implications of multinational production by looking at the ex-ante certainty equivalent of risky returns.[38] Second, in the financial autarky version of the model, we can revisit the so called quantity puzzle, that is, the fact that in standard international macromodels, consumption correlation generically exceeds output correlations, however in the data the opposite is the case. For that purpose I compare the cross-country correlation of consumption with the cross-country correlation of GDP in the financial autarky model when increasing FDI openness τ.

2.5.1 Risk sharing implications

In order to asses how FDI openness affects the riskiness of consumption and therefore welfare, I compute the certainty equivalent (in terms of consumption) that makes the households indifferent between the expected discounted value of consuming forever the expected value of consumption (and working forever one-third of the time) and the expected present value of the risky consumption and leisure stream at period 0. Formally, for all model versions, I compute λ that solves:

$$\frac{1}{1-\beta}U(E(C), E(L)) = E\left[\sum_{t=0}^{\infty}\beta^t U(C_t(1+\lambda), L_t)\right]$$

I will also refer to λ as the risk premium. Figure 2.5 shows the risk premium as defined above for the three model versions; the blue line refers to the complete market model with

[38]See Ramondo and Rappoport (2010) for theoretical results within a similar model with two periods.

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only country-specific shocks, the red-line refers to the complete market case with both country-specific and multinational-specific shocks, and the green short dashed line refers to the financial autarky model with both shocks. In all specifications the risk premium decreases with increasing FDI openness. For the complete markets model versions this means that more FDI linkages leads to a decrease in aggregate consumption risk. The first important thing to take away here is that the presence of technology capital has a decreasing effect on risk premia. Second, quantitatively, the model with just country-specific shocks (blue line) implies decrease of the conumption risk premium by 11 percent when comparing complete FDI openness ($\tau = 1$) to very low FDI openness ($\tau \to 0$).[39] In contrast, the complete market model with both shocks implies a decrease of 6 percent, a bit less than half of the effect without multinational-specific shocks. This means that multinational-specific shocks actually increase global risk and therefore mitigate the potential gains of multinational production in terms of aggregate consumption risk. This result goes into the same direaction as Kalemli-Ozcan, Papaioannou, and Perri (2013) who find that shocks to banking activity increase international co-movement.[40]

For completeness, in the financial autarky case, consumption risk premia fall by 15 percent. Note however, that this does not imply that aggregate risk goes up or down because FDI also increases cross-country risk sharing and here here only the net effect is plotted.

2.5.2 The puantity puzzle and international correlations

The standard one-good two-country international macro model with complete or incomplete markets predicts consumption correlation to be higher than GDP correlation. The fact that in the data for developed countries the opposite is true is known as the so called quantity puzzle. In this section I want to assess the impact of FDI openness on the relation between consumption correlation and GDP correlation. There have been various attempts to explain the quantity puzzle, for example shocks to household tastes (Stockman and Tesar, 1995) or incompleteness of international financial markets (Kehoe and Perri, 2002), but to the best of my knowledge these approaches do resolve the issue only partially. In addition, because of the special setup considered here, I can provide comparative static results with respect to FDI openness parameter τ. Of course, this exercise makes only sense in the financial autarky model (otherwise consumption correlation is always equal to one).

[39] Please note that here I am referring to relative changes of the risk premium not to changes in percentage points.

[40] Banks in their setup are the only firms who can invest abroad by giving loans to foreigners who do not have access to international financial markets

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Figure 2.6: FDI openness and real synchronization, financial autarky with both shocks

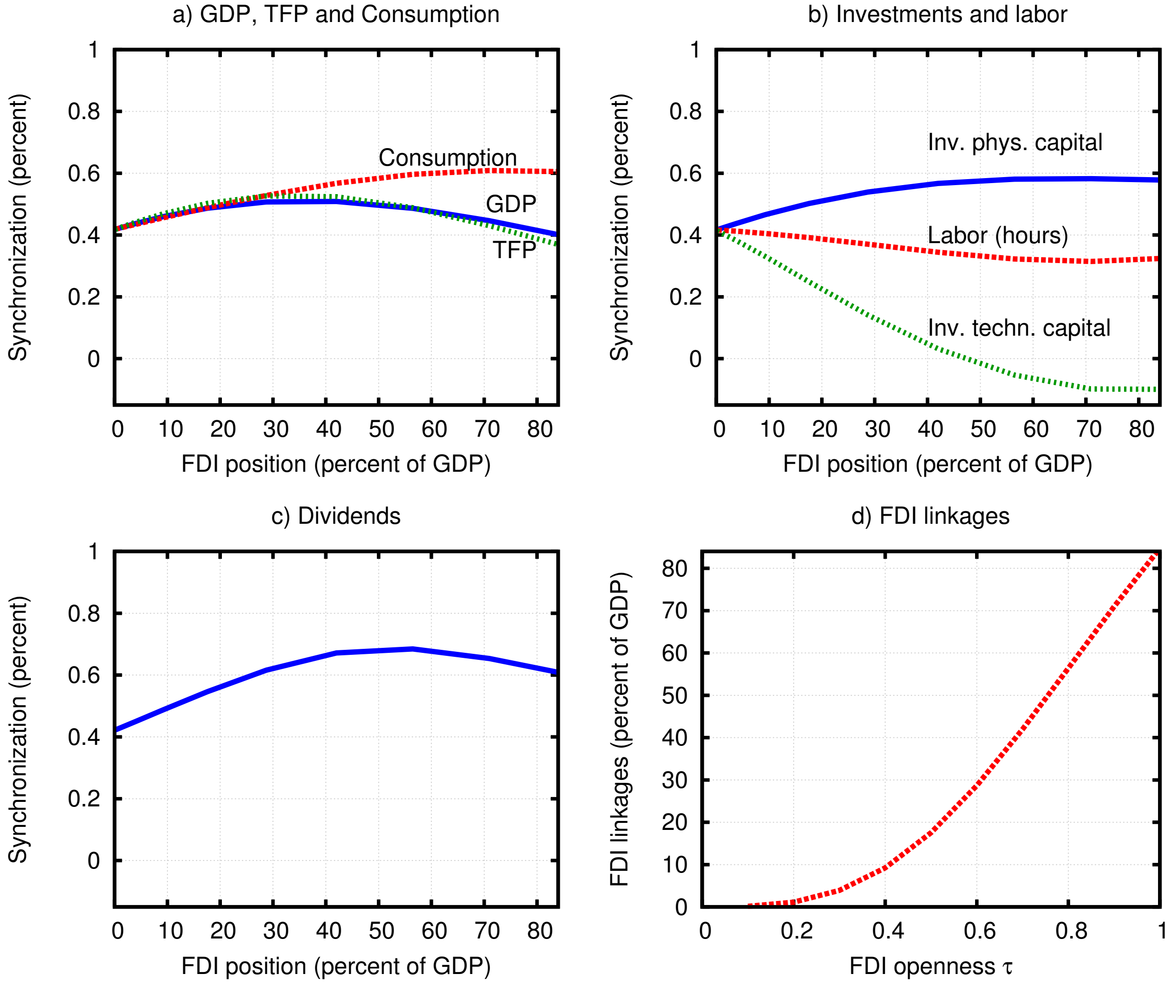

Figure 2.6 plots cross-country correlations of hp-filtered simulated quarterly data (using an HP parameter equal to 1600) for important real variables as a function of FDI integration (FDI/GDP) when varying τ smoothly from zero to one. Panel a) confronts the cross-country correlation of consumption (long dashed red line) with that of GDP (blue solid line) and TFP (short dashed green line). There are three noteworthy features here. First, for FDI integration below 25 percent, consumption correlation lies slightly below GDP correlation. Afterwards GDP correlation shows its hump-shaped pattern as already discussed above while consumption correlation continues to increase. Interestingly it does so very slowly and even when τ is equal to one (implying an FDI position relative to GDP of 85 percent) it is 0.6, way below perfect risk sharing. This is due to the presence of shocks to multinational-activity. Multinationals are able to hedge country-specific risks by efficiently allocating there resources across country borders; however they are still affected by multinational-specific shocks. Under the assumption that households cannot internationally trade any financial assets, they are not able to hedge away these risks.

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Therefore, one valid possibility to identify shocks to multinationals in this model, would be to match the relation between consumption correlation and GDP correlation. This is an interesting future research avenue.

The second interesting finding is evident from panel b) in figure 2.6. The cross-country correlation of investment to technology capital decreases and eventually becomes negative while investment in physical capital increases as FDI openness increases. In a sense, technology capital plays the role of capital in a standard business cycle model: The firms that are more efficient (high realisation of multinational-specific shock) increases its investment in technology capital while less productive firms find it too expensive to do so and therefore decrease their investment in technology capital.

Third, cross-country correlation of hours worked decreases when FDI openness increases in the financial autarky case considered here. There are two forces at work. On the one hand, labor correlation increases because of more FDI. The reason is the same as for investment in physical capital: foreign multinationals demand more labor when they have relatively high productivity; when countries are more open to FDI this affects more and more both countries and this potentially pushes up labor co-movement.[41] On the other hand, because of increasing risk sharing due to the presence of multinationals (they hedge country-specific shocks), households from countries with more efficient multinationals have a higher marginal utility of leisure and therefore work more. This pushes the cross-country correlation of hours

Table 2.8 summarizes the findings above but in a different way. It shows the same regressions as described in section 2.2 for investment and GDP also for consumption, labor, and TFP. In the column denoted by 'data (G7)' I report the empirical estimates using the empricial specification in (2.1) for the analogous synchronization measures for consumption, labor and TFP for the G7 countries. The remaining columns report the results of the same regressions for the three model versions. From the point estimates in the data column, the ranking of relation between FDI opennes and the respective variables should be the following way: Labor (hours), GDP, consumption, TFP. The complete market case with both shocks predicts the following ranking: Labor (hours), GDP, consumption, TFP (the consumption correlation is trivial as it always remains equal to one, of course). The interesting thing here is that it gets the ranking between Labor and GDP right. as indicated above, the financial autarky model predicts a point estimate for consumption that exceeds the estimate on consumption. Note however, if one increases the volatility of multinational-specific shocks, one could match the ranking

[41] In fact, under complete markets cross-country labor correlation unambigously increases as shown below in table 2.8. See discussion below.

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Table 2.8: Bilateral FDI linkages and synchronization: data vs. model., other real variables

	Data (G7)	Model		
		Complete Markets	Complete markets, both shocks	Financial autarky, both shocks
Coefficient on GDP synchronization	0.346 (1.41)	-0.136	0.253	0.081
Coefficient on consumption synchronization	0.196 (0.98)	0.000	0.000	0.140
Coefficient on labor synchronization	0.389 (1.75)	0.004	0.263	-0.019
Coefficient on tfp synchronization	0.13 (1.65)	-0.153	-0.007	0.060

Notes: The table reports the estimated coefficients in the three model versions. For convenience, column one reports the estimated coefficients obtained from the data, taken from column (3) in tables 2.2 and 2.3, respectively. For the empirical estimates, t-values are reported in parenthesis below. The last three columns report the estimated coefficient of the three model versions. In the first row, the dependent variable is investment synchronization, in the second row GDP synchronization, and in the third row it is actual value added (Y). The right hand side variable in all regressions is log of the sum of bilateral FDI positions divided by the sum of the countries' GDP.

of the estimates.[42]

2.6 Summary

In this paper, I document that FDI integration and investment synchronization are positively correlated. This is an important finding because the standard international business cycle model predicts the opposite. I then propose a tractable international business cycle model where multinational firms engage in FDI and multinational activity is subject to shocks. I show that the positive association between FDI openness and investment synchronization is consistent with the hypothesis that multinationals play an important role for the international transmission of shocks. The model also gives a rationale for the weak link between financial integration on GDP co-movement. Due to mismeasurement of GDP reported in national accounts, actual output co-movement is overestimated when

[42]In a model version with only multinational-specific shocks, GDP correlation generically exceeds consumption correlation. Only for values of τ very close to one the ranking changes. These are not shown but available upon request.

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countries are relatively closed and underestimated when countries are relatively open to FDI.

There are three main lessons from the theory. First, more FDI openness leads unambiguously to higher investment synchronization across country borders. This result holds in model specifications with complete and financial autarky, respectively.

Second, shocks to multinational activity are important to quantitatively account for the estimated link between FDI integration and investment synchronization. The regression coefficients estimated on artificial data from the model with multinational-specific shocks quantitatively explain 20 percent of the empirical regression coefficients, compared to 6 percent in the model version without multinational-specific shocks.

Third, because measured GDP is distorted, the regression coefficients on actual output synchronization are 0.04 to 0.1 percentage points higher than the coefficients for GDP synchronization. This are significant numbers amounting to 30 to 75 percent of the empirical point estimate. As I abstract from many other forms of intangible capital (potentially worsen the measurement issue) it is not clear whether these estimates are upper or lower bounds. I leave this for future research.

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Appendix A

Appendix to Chapter 1

A.1 Data

The following series used in Figure 1 and Figure 2 are from Federal Reserve Economic Data: the federal funds rate, the one year mortgage interest rate (released by the Primary Mortgage Market Survey by Freddie Mac), the mortgage (defined as home mortgages from the balance sheet of U.S. households and nonprofit organizations) and real estate (defined as the market value of real estate from the balance sheet of U.S. households and nonprofit organizations). All series are at quarterly frequency. The series for house prices is the National Composite Home Price Index for the United States (the release is by S&P/Case-Shiller Home Price Indices). The spread has been calculated as the difference between the one year mortgage interest rate and the federal funds rate each quarter.

In the calibration section, we calculate housing wealth as percentage of US nominal GDP (yearly) by using historical data of the flows of funds tables from the Board of Governors. US nominal GDP is from the Bureau of Economic Analysis. Our definition of housing wealth includes the market value of real estate belonging to households, non-profit and non-financial non-corporate business.

The micro-data used for the calibration of the relative wealth distribution of borrowers and the leverage ratio are provided by the 1998 to 2009 waves of Survey of Consumer Finances (SCF). Unfortunately, the SCF does not provide information on the precise date at which households were interviewed. Consequently, we assume that the observed portfolios in 2009 reflect the distribution of household net worth at the end of 2007. Averaging for all the waves between 1998 and 2009 helps in targeting data moments that are not strongly influenced by the years preceding the Great Recession. Surveyed households have been partitioned into *borrowers* and *savers* depending on their net asset

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position. The net asset position is defined as the sum of savings bonds, directly held bonds, the cash value of life insurance, certificates of deposits, quasi-liquid retirement accounts and all other types of transaction accounts (we consider these aggregated values to be deposits in the model) minus the debt secured by primary residence (mortgages, home equity loans, etc.) and the debt secured by other residential property, credit card debt and other forms of debt (we refer to these aggregated values as debt in the model). If the net asset position is positive, we consider the household to be a saver in our model economy, otherwise we consider her to be a borrower. The reason to use a broad definition of aggregate deposits and debt in the data counterpart is that it is difficult to target borrowers and savers by strictly restricting attention to particular classes of debt. We moreover define net wealth per capita as the sum of the net asset position and the value of the primary residence and other residential properties, for both leveraged and net savers. Finally, we aggregate the net wealth of both groups (borrowers and savers) and we calculate the relative net wealth of borrowers as the ratio between their net wealth over the total net wealth in the economy. The leverage ratio of the borrowers is instead obtained as the weighted average mean (using SCF sample weights) of the net asset position over the value of primary and secondary residences. The reference values that are matched by the model are obtained by cutting the 5% tails of the distribution of net worth in each wave of the SCF This is done to cut the extreme observations that may bias the average values of net worth in the US economy. We want, in fact, to avoid the possibility of including in the range of borrower households that maintain large positions in the stock or housing markets and hold little savings.

A.2 Numerical Details

The algorithm employed is an adoption of the time-iteration procedure with linear interpolation used in Grill and Brumm (2010). As we have only two agents, a fine grid for wealth is enough to deliver satisfactorily small Euler errors. For this reason, we do not adapt the grid around the points where the collateral constraint is binding, as proposed by Grill and Brumm (2010).

A.2.1 Equilibrium conditions

We want to describe the equilibrium in our economy in terms of policy functions that map the current state into current policies. Furthermore, we want to focus on recursive mappings - that is, time-invariant functions that satisfy the period-by-period first-order equilibrium conditions. In what follows, we characterize these equilibrium conditions in

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every detail. For each agent $i = b, s$, denote by $\nu_i(w, z)$ the Lagrange multiplier with respect to her budget constraint and by $\phi_i(w, z)$ the Kuhn- Tucker multiplier attached to her collateral constraint. In addition, we treat saving and debt as two separate assets: saving is an asset in which the agent can only take long positions, $s_i \geq 0$; debt is an asset with return R_D in which agents can only take short positions, $d_i \leq 0$. Denote the Kuhn-Tucker multipliers attached to these inequalities as χ_i and μ_i, respectively. Then, for each tuple consisting of wealth and exogenous state today $\sigma = (w, z)$, the (time-invariant) policy and pricing functions have to satisfy the following system of equations (we will show below how to solve for these time-invariant functions):

- Agent's first order conditions

$$\begin{aligned}
u_1(c_i(\sigma), h_i(\sigma)) - \nu_i(\sigma) &= 0 \\
u_2(c_i(\sigma), h_i(\sigma)) - q(\sigma)\nu_i(\sigma) &= 0 \\
-\nu_i(\sigma) + \beta^i E[\nu_i(\sigma^+)|\sigma]R(\sigma) + \chi_i(\sigma) &= 0, \qquad i = s, b \\
-\nu_i(\sigma) + \beta^i E[\nu_i(\sigma^+)|\sigma]R_D + \phi_i(\sigma)R_D(w, z) - \mu_i(\sigma) &= 0 \\
-\nu_i(\sigma)q(\sigma) + u_2(c_i(\sigma), h_i(\sigma)) + & \\
+\beta^i E[\nu_i(\sigma^+)q(\sigma^+)]|\sigma] + \phi_i(\sigma)mE[q(\sigma^+)|\sigma] &= 0
\end{aligned}$$

- Agent's budget constraints

$$\begin{aligned}
n_b y(s) + n_b \Upsilon(\sigma) + w \cdot q(\sigma) - d_b(\sigma) - s_b(\sigma) - q(\sigma)h_b(\sigma) - c_b(\sigma) &= 0 \\
n_s y(s) + n_s \Upsilon(\sigma) + (1 - w) \cdot q(\sigma) - d_s(\sigma) - s_s(\sigma) - q(\sigma)h_s(\sigma) - c_s(\sigma) &= 0
\end{aligned}$$

  NB: Here we have already used the definition for the borrower's wealth share and rewritten the budget constraints in these terms (see the law of motion for wealth below as a reminder of how we defined the wealth share).

- Zero profits in the financial sector

$$\theta(s) \cdot R_D(\sigma) - R(\sigma) = 0$$

- Market clearing in housing and financial sector

$$\begin{aligned}
h_s(\sigma) + h_b(\sigma) - 1 &= 0 \\
d_b(\sigma) + d_s(\sigma) + \theta(s) \cdot (s_b(\sigma) + s_s(\sigma)) &= 0
\end{aligned}$$

- Transfers

$$\Upsilon(\sigma) - (1 - \theta(s))(s_b(\sigma) + s_s(\sigma)) = 0$$

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- Complementary slackness conditions

$$\begin{aligned}\mu_i(\sigma) \geq 0, d_i(\sigma) \geq 0, &\quad \mu_i(\sigma) \perp d_i(\sigma) \\ \chi_i(\sigma) \geq 0, s_i(\sigma) \geq 0, &\quad \chi(\sigma) \perp s_i(\sigma), \qquad i = s, b \\ \phi_i(\sigma) \geq 0, CC_i(\sigma) \geq 0, &\quad \phi_i(\sigma) \perp CC_i(\sigma)\end{aligned}$$

where $CC_i(\cdot)$ is the collateral constraint of agent i, that is,

$$CC_i(\sigma) \equiv R_D(\sigma) d_i(\sigma) + mE[q(\sigma^+)|\sigma] h_i(\sigma) \geq 0$$

- Implicit "Law of motion" for borrower's wealth share

$$w^+(\sigma, z^+) \equiv \frac{R_D(\sigma) d_b(\sigma) + R(\sigma) s_b(\sigma) + q(w^+(\sigma, z^+), z^+) h_b(\sigma)}{q(w^+(\sigma, z^+), z^+)}.$$

A.2.2 Algorithm

The structure of the above period-by-period equilibrium conditions can be summarized as follows: Given a guess for the policy and pricing functions in the next period - denoted by f^{prime} - we can compute the expectations in the agents' first order conditions. The functions that map current states to current policies - denoted by f - are then obtained by solving the static system of non-linear given in the previous subsection. More formally, the structure of the problem can be summarized as follows. For all tuples $\sigma = (w, z)$, we have

$$\psi(f^{prime})(\sigma, f(\sigma), \mu(\sigma)) = 0, \qquad \zeta(\sigma, f(\sigma)) \geq 0 \perp \mu(\sigma) \geq 0.$$

The system of equations $\psi[f^{prime}](\cdot)$ contains first order conditions of agents and the financial sector and market clearing conditions. The function $\zeta(\cdot)$ contains the sign restrictions and collateral constraints. $\mu(\cdot)$ denotes the respective Kuhn-Tucker multipliers. A recursive policy function f then solves $\psi[f](\sigma, f(\sigma)\mu(\sigma)) = 0$ such that the complementary slackness conditions are satisfied. The time iteration algorithm defined below finds the approximate recursive policy function iteratively.

In each iteration, taking as given a guess for f^{prime}, we obtain f by solving the above system of equations and then updating our guess by interpolating the obtained policy function on the implicitly defined next period wealth. The following box summarizes our algorithm in a form of Pseudo-code:

1. Select a grid $\mathcal{W}$, an initial guess f^{init} and an error tolerance ϵ. Set $f^{prime} = f^{init}$.

2. Make one time-iteration step:

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(a) For all $\sigma = (w, z)$, where $w \in \mathcal{W}$, find the function $f(\sigma)$ that solves

$$\psi(f^{prime})(\sigma, f(\sigma), \mu(\sigma)) = 0, \qquad \zeta(\sigma, f(\sigma)) \geq 0 \perp \mu(\sigma) \geq 0.$$

(b) Use the solution f and the guess f^{prime} to update wealth tomorrow and interpolate f on the obtained values for wealth tomorrow.

3. If $||f - f^{prime}|| < \epsilon$, go to step 4. Else set $f^{prime} = f$ and repeat step 2.

4. Set numerical solution $\tilde{f}$ equal to the solution of the infinite horizon problem, $\tilde{f} = f$.

A.2.3 Kuhn-Tucker equations (Garcia-Zangwill trick)

At each grid point - given the guesses of the policy functions for the next period - we have to solve a system of nonlinear equations, containing both inequalities and equalities. The period-by-period equilibrium conditions are basically standard Kuhn-Tucker (K-T) conditions. In order to employ standard non-linear equation solvers like fsolve in Matlab or Ziena's Knitro, it is computationally more stable to eliminate the inequalities and recast the problem as a system consisting of equations only. In this section we describe how to do this. In general, we can write the Kuhn-Tucker conditions of any convex NLP problem as:

$$\begin{aligned} \Delta f(x)' + \sum_{j=1}^{r} \lambda_j \Delta g_j(x)' + \sum_{j=1}^{s} \mu_j \Delta h_j(x)' = 0 \\ \lambda_j \geq 0, g_j(x) \geq 0, \qquad j = 1, \dots, r \\ \lambda_j g_j(x) = 0, \qquad j = 1, \dots, r \\ h_j(x) = 0, \qquad j = 1, \dots, s \end{aligned} \tag{A.1}$$

plus a constraint qualification restriction (CQ). The system in (A.1) are mixtures of equalities and inequalities. Since inequalities tend to be cumbersome and can potentially prevent numerical software from solving the NLP via path-following, we will rewrite the K-T conditions so that they are a system consisting solely of equations (Zangwill and Garcia, 1981). The reformulation is as follows. Let k be a positive integer, and given $\alpha \in \mathbb{R}^1$, define:

$$\begin{aligned} \alpha^+ &= [\max\{0, \alpha\}]^k \\ \alpha^- &= [\max\{0, -\alpha\}]^k. \end{aligned}$$

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Hence, we always have $\alpha^+ \geq 0$, $\alpha^- \geq 0$, and $\alpha^+\alpha^- = 0$. Note also that both variables, $\alpha^+$and α^-, are $(k-1)$-continuously differentiable. Using this transformation, we can recast the K-T conditions and create the Kuhn-Tucker *equations* (Zangwill and Garcia, 1981):

$$\begin{gathered}\Delta f(x)' + \sum_{j=1}^{r} \alpha_j^+ \Delta g_j(x)' + \sum_{j=1}^{s} \mu_j \Delta h_j(x)' = 0 \\ \alpha_j^- - g_j(x) = 0, \qquad j = 1, \ldots, r \\ h_j(x) = 0, \qquad j = 1, \ldots, s\end{gathered} \tag{A.2}$$

where $\alpha = (\alpha_1, \ldots, \alpha_r) \in R^r$ and (α^+, α^-) are defined as above. Note that the (K-T) equations defined here are precisely equivalent to the K-T conditions in (A.1). In particular, if (x^*, α^*, μ^*) satisfies the K-T equations, then (x^*, λ^*, μ^*) satisfies the the K-T conditions with $\lambda_j^* \equiv (\alpha_j^*)^+$, $j = 1, \ldots, r$. Conversely, if (x^*, λ^*, μ^*) satisfies the K-T conditions in (A.1), then (x^*, α^*, μ^*) satisfies the K-T equations in (A.2) with

$$\alpha_j^* \equiv \begin{cases} (\lambda_j^*)^{1/k} & \text{if } g_j(x^*) = 0 \\ -(g(x^*)_j)^{1/k} & \text{if } g_j(x^*) > 0 \end{cases} \qquad j = 1, \ldots, r.$$

A.2.4 Numerical Accuracy

In order to measure the accuracy of our approximation procedure, we calculate two statistics: first, we compute the relative Euler errors along the equilibrium path for very long time series. Second, for each exogenous shock, we randomly draw 3000 points from the wealth grid and compute the relative Euler Errors. To summarize the findings: for all simulated models, the maximum relative Euler Error is 3e-5 (or -4.5 in log(10)-scale). This implies that an agent, using our approximation of the equilibrium policy functions, would lose 30 Dollars for each million spent. It is important to compare this number to the welfare gains we obtain in the benchmark model. The borrowers' welfare loss on impact of an financial intermediation shock is 0.07 percentage points, that is, in log(10) scale, equal to -3.15. This number is one order of magnitude bigger, so even when netting these numbers by the mistakes that agents make, we conclude that our quantitative findings are still valid.

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Appendix B

Appendix to Chapter 2

B.1 Data

Quarterly output and investment data are from OECD Quarterly National Accounts. The real series are annualized in constant US Dollars (OECD reference year), converted at fixed PPP exchange rates and seasonally adjusted (series VPVOBARSA). Correlations are the yearly average of 20 quarter rolling window estimates after HP-filtering data. The synchronization measures are calculated as explained in the main text. Yearly nominal GDP is from OECD National Accounts, annualized values in current US Dollars, converted at current PPP exchange rates, and seasonally adjusted (series CPCARSA). Bilateral outward and inward foreign direct investment data are from the OECD Foreign Direct Investment Database at a yearly frequency. Note that a general caveat of FDI data is that it is usually recorded at historical cost, so the FDI positions do not necessarily reflect actual market values. Total foreign direct investment data are from the OECD foreign direct investment data and from Lane and Milesi-Ferretti (2007). Bilateral exports and imports are from the OECD monthly trade statistics. Population data are from OECD labor market statistics. Industrial specialization in year t is calculated as in Imbs (2006); it is the absolute distance of the shares in value added of each sector in countries i and j, summing over all sectors. The data are from the OECD Statistics on Measuring Globalisation.

Table B.1 summarizes the country pairs with all FDI data are available throughout the whole sample from 1990 to 2006 and 1995 to 2006, respectively. An exception is Japan, bilateral FDI data are available from 1985 to 1994 and 1996 to 2006 only, 1995 is not reported in the OECD database. For not deleting a G7 member from the analysis, I used linear interpolation to obtain the bilateral FDI positions for 1995.

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Table B.1: Country pairs used in empirical analysis

Balanced Sample, 1991 - 2006

AUT,CAN	AUT,DEU	AUT,FRA	AUT,GBR	AUT,USA	CAN,DEU	CAN,FRA	CAN,GBR
CAN,ITA	CAN,JPN	CAN,NOR	CAN,USA	DEU,GBR	DEU,ITA	DEU,JPN	DEU,NLD
DEU,NOR	DEU,SWE	DEU,USA	FRA,DEU	FRA,GBR	FRA,ITA	FRA,JPN	FRA,NLD
FRA,NOR	FRA,SWE	FRA,USA	GBR,USA	ITA,GBR	ITA,JPN	ITA,NLD	ITA,USA
JPN,GBR	JPN,NLD	JPN,USA	NLD,GBR	NLD,USA	NOR,SWE	NOR,USA	SWE,USA

Notes: The table lists the country-pairs for which bilateral data were available in all years without gaps used in the respective samples. Essentially, it consists of the 21 G7 country pairs plus pairs involving Austria, Netherlands, Norway, and Sweden. Please read the note regarding the bilateral FDI positions reported by Japan in this appendix.

B.2 Additional empirical results and robustness checks

This section presents several robustness checks for the empirical results in the main text. First, I include in the analysis all 40 countries for which I have information between the years 1991 to 2006. Table B.2 reports the results for investment synchronization. The main message remains, more bilateral FDI linkages are associated with more investment synchronization. The point estimates are of the same order of magnitude and do not differ between G7 country pairs and other country pairs (I tested this formally by including an interaction term between a G7-pair-dummy and FDI and trade openness, respectively. The estimate is not significant different from zero. The results are not reported here but available upon request from the author). Note that the estimates significant different from zero for specifications in which interaction terms between FDI openness and industrial specialization and between trade openness and industrial specialization, respectively, are included. For that purpose I define a dummy variable that takes the value 0 if specialisation is smaller or equal to the median specialization. For country-years in which specialization exceeds the median specialization, the dummy is equal to one.

The results for GDP synchronization are as for G7 countries only. The point estimates are positive but not statistically different from zero, even when including interaction terms between FDI openness and specialization and trade openness and specialization.

Second, in table B.4 and B.5, I report regressions for HP-filtered data. For this purpose, I regress the 20 quarter cross-correlation of HP-filtered data in period t on FDI and trade openness in year $t-5$, that is, the initial value of the 20-quarter window. All other controls (log of product of GDP per capita, log of product of population, log of industrial specialization index) are also lagged by 5 years. Note that qualitatively the results remain the same when regressing the correlations on the 5 year averages of the time windows (not reported, available upon request). I prefer the former specification because there is more likelihood that exogeneity of the regressors and the residuals hold. For the HP-filtered data, I also splitted the sample in three non-overlapping time periods, 1991- 1996,

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Table B.2: Bilateral FDI linkages and investment synchronization, all country pairs

Dependent Variable: Investment growth synchronization (annualized)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
FDI/GDP	0.676	0.537	1.201†	2.044**				
	(0.83)	(0.84)	(1.58)	(2.65)				
$\frac{FDI}{GDP} \times Dspec$				-0.978**				
				(-2.63)				
FDI/Total FDI					0.545	0.146	0.814	1.239*
					(0.69)	(0.21)	(1.12)	(1.78)
$\frac{FDI}{TotalFDI} \times Dspec$								-0.420
								(-0.98)
Country-pair fixed	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time fixed	No	Yes	Yes	Yes	No	Yes	Yes	Yes
Country trends	Yes	No	Yes	Yes	Yes	No	Yes	Yes
R-squared (within)	0.171	0.187	0.251	0.258	0.172	0.183	0.249	0.250
Observations	600	600	600	600	600	600	600	600

Notes: The table reports panel (country-pair) fixed-effect coefficients estimated over the period 1991 to 2006 of the G7 countries plus country pairs involving Austria, Netherlands, Norway, and Sweden (in total, 40 country pairs). These are all countries for which bilateral FDI positions are available for all years. The dependent variable is minus one times the absolute value of the difference in quarterly growth rate of aggregate investment between country i and j in year t (the yearly estimate is obtained by averaging over the respective four quarterly estimates). In columns (1) - (4) FDI integration is measured by the log of the share of the stock of bilateral Foreign Direct Investment Assets and Liabilities between countries i and j in the previous year relative to the sum of the two countries' GDP in the previous year (denoted FDI/GDP). In columns (5) - (8) FDI integration is measured by the log of the share of the stock of bilateral Foreign Direct Investment Assets and Liabilities between countries i and j in the previous year relative to the sum of the two countries' total FDI Assets and FDI Liabilities in the entire world in the previous year (denoted FDI/Total FDI). All specifications also include the log of the two countries' GDP per capita, the log of the product of both countries' populations in the previous year, trade openness measured by Trade/GDP (in spec. (1) to (4)) or Trade/Total Trade (in spec. (5) to (8)), and the log of an index measuring industrial specialization, all lagged by one period (year). The specification in (4) includes an interaction term between FDI/GDP and a dummy variable (Dspec) that takes on the value one if the specialization index of a country pair is bigger than the median specialization of the sample and zero otherwise. Analogously it includes the same interaction term for Trade/GDP (not reported). Specification (8) includes an interaction term between FDI/Total FDI and a dummy variable (Dspec) that takes on the value one if the specialization index of a country pair is bigger than the median specialization of the sample and zero otherwise. The specifications in columns (1) and (5) include country-specific linear time-trends. The specifications in columns (2) and (6) include time fixed-effects. The specifications in columns (3), (4), (7), and (8) include time fixed-effects and country-specific linear time-trends. Standard errors adjusted for panel (country-pair) specific auto-correlation and heteroskedasticity and corresponding t-statistics are reported below the estimated coefficients. † denotes significance at the 85% confidence level, * denotes significance at the 90% confidence level, ** denotes significance at the 95% confidence level, *** denotes significance at the 99% confidence level. For a detailed data description see appendix B.1.

1996- 2001, and 2001 - 2006. As dependent variables I use the cross-country correlation of hp-filtered GDP and investment, respectively, computed for the 20 quarters of each time period. I then regress GDP and investment correlation on the log of bilateral FDI linkages at the beginning of each time period, that is, FDI position in 1991, 1996, and 2001, respectively. For all these robustness checks, the findings are similar to the ones of the benchmark estimates in the main text. More FDI linkages are associated with higher investment correlations, and there is no statistical link between FDI linkages and GDP correlations (though the point estimate is positive).

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Table B.3: Bilateral FDI linkages and GDP synchronization, all country-pairs

Dependent Variable: GDP growth synchronization (annualized)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
FDI/GDP	0.393*	-0.00377	0.297†	0.269				
	(1.88)	(-0.02)	(1.48)	(1.21)				
$\frac{FDI}{GDP} \times Dspec$				0.0298				
				(0.25)				
FDI/Total FDI					0.333†	-0.0497	0.290	0.209
					(1.55)	(-0.24)	(1.46)	(0.94)
$\frac{FDI}{TotalFDI} \times Dspec$								0.0799
								(0.60)
Country-pair fixed	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time fixed	No	Yes	Yes	Yes	No	Yes	Yes	Yes
Country trends	Yes	No	Yes	Yes	Yes	No	Yes	Yes
R-squared (within)	0.239	0.279	0.360	0.360	0.234	0.278	0.360	0.361
Observations	600	600	600	600	600	600	600	600

Notes: The table reports panel (country-pair) fixed-effect coefficients estimated over the period 1991 to 2006 of the G7 countries plus country pairs involving Austria, Netherlands, Norway, and Sweden (in total, 40 country pairs). These are all countries for which bilateral FDI positions are available for all years. The dependent variable is minus one times the absolute value of the difference in quarterly growth rate of aggregate investment between country i and j in year t (the yearly estimate is obtained by averaging over the respective four quarterly estimates). In columns (1) - (4) FDI integration is measured by the log of the share of the stock of bilateral Foreign Direct Investment Assets and Liabilities between countries i and j in the previous year relative to the sum of the two countries' GDP in the previous year (denoted FDI/GDP). In columns (5) - (8) FDI integration is measured by the log of the share of the stock of bilateral Foreign Direct Investment Assets and Liabilities between countries i and j in the previous year relative to the sum of the two countries' total FDI Assets and FDI Liabilities in the entire world in the previous year (denoted FDI/Total FDI). All specifications also include the log of the two countries' GDP per capita, the log of the product of both countries' populations in the previous year, trade openness measured by Trade/GDP (in spec. (1) to (4)) or Trade/Total Trade (in spec. (5) to (8)), and the log of an index measuring industrial specialization, all lagged by one period (year). The specification in (4) includes an interaction term between FDI/GDP and a dummy variable (Dspec) that takes on the value one if the specialization index of a country pair is bigger than the median specialization of the sample and zero otherwise. Analogously it includes the same interaction term for Trade/GDP (not reported). Specification (8) includes an interaction term between FDI/Total FDI and a dummy variable (Dspec) that takes on the value one if the specialization index of a country pair is bigger than the median specialization of the sample and zero otherwise. The specifications in columns (1) and (5) include country-specific linear time-trends. The specifications in columns (2) and (6) include time fixed-effects. The specifications in columns (3), (4), (7), and (8) include time fixed-effects and country-specific linear time-trends. Standard errors adjusted for panel (country-pair) specific auto-correlation and heteroskedasticity and corresponding t-statistics are reported below the estimated coefficients. † denotes significance at the 85% confidence level, * denotes significance at the 90% confidence level, ** denotes significance at the 95% confidence level, *** denotes significance at the 99% confidence level. For a detailed data description see appendix B.1.

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Table B.4: Bilateral FDI linkages and investment correlations, hp-filtered data

Dependent Variable: cross-country correlation of hp-filtered investment								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
FDI/GDP	0.230**	0.248***	0.214**	0.237**				
	(2.38)	(2.88)	(2.31)	(2.50)				
Trade/GDP				-0.594**				
				(-2.27)				
FDI/Total FDI					0.167†	0.193**	0.188*	0.243**
					(1.66)	(2.09)	(1.99)	(2.50)
Trade/Total Trade								-0.776***
								(-3.77)
Country-pair fixed	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time fixed	No	Yes	Yes	Yes	No	Yes	Yes	Yes
Country trends	Yes	No	Yes	Yes	Yes	No	Yes	Yes
R-squared (within)	0.396	0.367	0.461	0.475	0.388	0.357	0.459	0.490
Country-pairs	40	40	40	40	40	40	40	40
Observations	480	480	480	480	480	480	480	480

Notes: The table reports panel (country-pair) fixed-effect coefficients estimated over the period 1991 - 2006, using a balanced panel for 40 country-pairs for which bilateral FDI positions are available for all years. A list of included countries-pairs can be found in table B.1. The dependent variable is the five-year average cross-country correlation of hp-filtered investment in country i and j averaged over the past 20 quarters. All right hand-side variables are lagged by five periods (years), that is, measured at the beginning of the rolling windo. In columns (1) - (4) FDI integration is measured by the log of the share of the stock of bilateral Foreign Direct Investment Assets and Liabilities between countries i and j in the initial year of the respective time period relative to the sum of the two countries' GDP in the in the initial year of the respective time period (denoted FDI/GDP). In columns (5) - (8) FDI integration is measured by the log of the share of the stock of bilateral Foreign Direct Investment Assets and Liabilities between countries i and j in the initial year of the respective time period relative to the sum of the two countries' total FDI Assets and FDI Liabilities vis-a-vis the entire world in the initial year of the respective time period (denoted FDI/Total FDI). All specifications also include the log of the two countries' GDP per capita, the log of the product of the two countries' population in the initial year of the respective time period, and the log of industrial specialization index. Specification (4) includes the log of the share of bilateral export and import flows between countries i and j in the initial year of the respective time period relative to the sum of the two countries' GDP in the initial year of the respective time period (Trade/GDP). Specification (8) includes the log of the share of bilateral export and import flows between countries i and j in the initial year of the respective time period relative to the sum of the two countries' total exports and imports in the initial year of the respective time period (Trade/Total Trade). The specifications in columns (1) and (5) include country-specific linear time-trends. The specifications in columns (2) and (6) include time fixed-effects. The specifications in columns (3), (4), (7), and (8) include time fixed-effects and country-specific linear time-trends. Standard errors adjusted for panel (country-pair) specific auto-correlation and heteroskedasticity and corresponding t-statistics are reported below. † denotes significance at the 85% confidence level, * denotes significance at the 90% confidence level, ** denotes significance at the 95% confidence level, *** denotes significance at the 99% confidence level. For a detailed data description see appendix B.1.

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Table B.5: Bilateral FDI linkages and output correlations, hp-filtered data

Dependent Variable: cross-country correlation of hp-filtered GDP								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
FDI/GDP	-0.0267	0.0835	0.0299	0.0410				
	(-0.26)	(0.85)	(0.37)	(0.49)				
Trade/GDP				-0.282				
				(-1.27)				
FDI/Total FDI					-0.0114	0.122	0.0772	0.113
					(-0.11)	(1.39)	(0.96)	(1.29)
Trade/Total Trade								-0.505***
								(-2.80)
Country-pair fixed	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time fixed	No	Yes	Yes	Yes	No	Yes	Yes	Yes
Country trends	Yes	No	Yes	Yes	Yes	No	Yes	Yes
R-squared (within)	0.297	0.399	0.515	0.518	0.297	0.402	0.517	0.530
Country-pairs	40	40	40	40	40	40	40	40
Observations	480	480	480	480	480	480	480	480

Notes: The table reports panel (country-pair) fixed-effect coefficients estimated over the period 1991 - 2006, using a balanced panel for 40 country-pairs for which bilateral FDI positions are available for all years. A list of included countries-pairs can be found in table B.1. The dependent variable is the five-year average cross-country correlation of hp-filtered GDP in country i and j averaged over the past 20 quarters. All right hand-side variables are lagged by five periods (years), that is, measured at the beginning of the rolling windo. In columns (1) - (4) FDI integration is measured by the log of the share of the stock of bilateral Foreign Direct Investment Assets and Liabilities between countries i and j in the initial year of the respective time period relative to the sum of the two countries' GDP in the in the initial year of the respective time period (denoted FDI/GDP). In columns (5) - (8) FDI integration is measured by the log of the share of the stock of bilateral Foreign Direct Investment Assets and Liabilities between countries i and j in the initial year of the respective time period relative to the sum of the two countries' total FDI Assets and FDI Liabilities vis-a-vis the entire world in the initial year of the respective time period (denoted FDI/Total FDI). All specifications also include the log of the two countries' GDP per capita, the log of the product of the two countries' population in the initial year of the respective time period, and the log of industrial specialization index. Specification (4) includes the log of the share of bilateral export and import flows between countries i and j in the initial year of the respective time period relative to the sum of the two countries' GDP in the initial year of the respective time period (Trade/GDP). Specification (8) includes the log of the share of bilateral export and import flows between countries i and j in the initial year of the respective time period relative to the sum of the two countries' total exports and imports in the initial year of the respective time period (Trade/Total Trade). The specifications in columns (1) and (5) include country-specific linear time-trends. The specifications in columns (2) and (6) include time fixed-effects. The specifications in columns (3), (4), (7), and (8) include time fixed-effects and country-specific linear time-trends. Standard errors adjusted for panel (country-pair) specific auto-correlation and heteroskedasticity and corresponding t-statistics are reported below. † denotes significance at the 85% confidence level, * denotes significance at the 90% confidence level, ** denotes significance at the 95% confidence level, *** denotes significance at the 99% confidence level. For a detailed data description see appendix B.1.

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B.2.1 The financial autarky model with country-specific shocks only

This appendix presents - for completeness - the results on the financial autarky model and country-specific productivity shocks only. The purpose is to show that the main results on investment and GDP synchronization remain unaltered, even when abstracting from multinational-specific shocks under financial autarky. Figure B.1 shows a similar plot with respect to investment synchronization as in the main text. In panel a), I show the benchmark results on the complete markets model with both shocks. Panel b) is new and shows the financial autarky model with productivity shocks only, panel c) and d) showing investment synchronization and FDI positions as functions of FDI openness parameter τ for the financial autarky economy with both shocks, hence repeating what is presented in the main text. It is evident from panel b) that the presence of technology capital only leads to the prediction of increased investment synchronization, even when financial markets are shut down and we abstract from shocks to multinational activity. The mechanisms behind this finding are exactly the same as outlined in the main text for the complete financial markets economies: When countries are more open to FDI, the returns of technology capital increase for both domestic and foreign multinationals, hence investment in technology capital increases in both countries. Because returns on all capital types are equalized within firms, multinationals increase investment in physical capital both at home and abroad, leading to an increase in investment co-movement.

Regarding GDP co-movement, figure B.2 shows that the findings under financial autarky with productivity shocks only are very similar to the complete market model with productivity only.

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Figure B.1: FDI openness and Investment synchronization, details on the financial autarky model

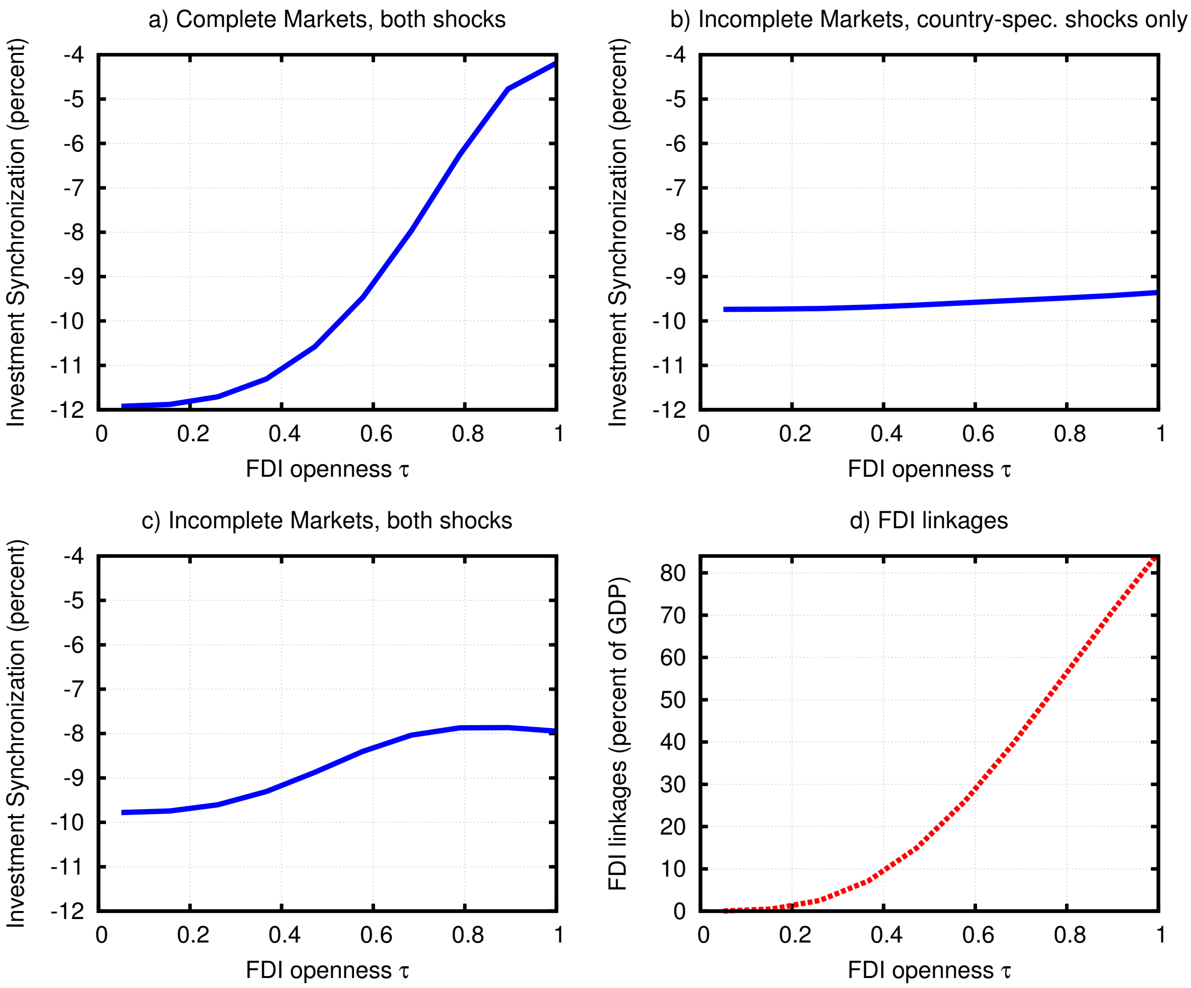

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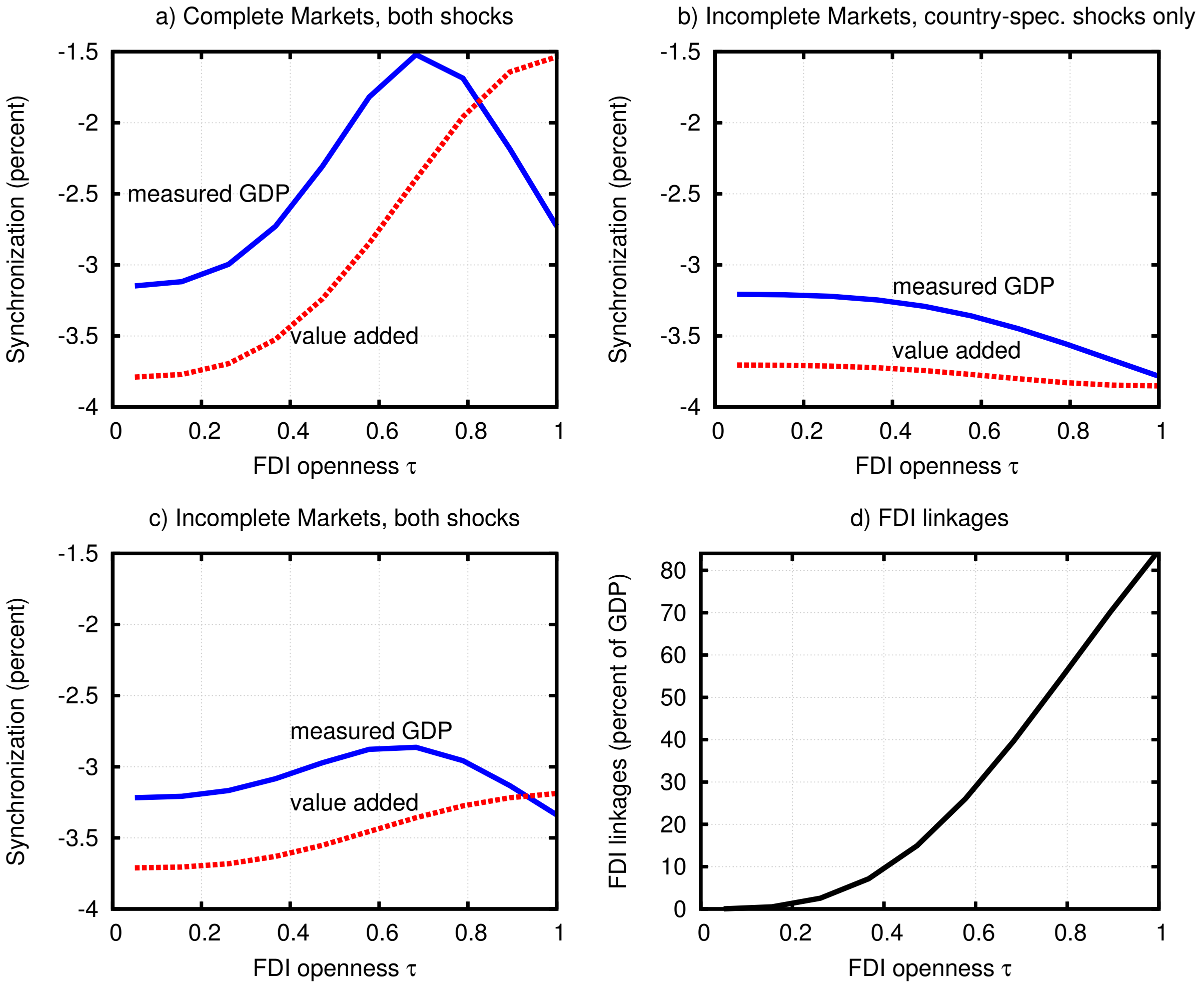
Figure B.2: FDI openness and GDP synchronization, details on the financial autarky model



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B.3 Model details and additional results for the financial autarky case

B.3.1 Model Equations

In this subsection, I list the model equations, including the first order conditions of households and firms. Note that because the model is symmetric, the first order conditions for foreign firms are omitted.

Output in tradable good sector in the home country and relative price of intermediate good bundle

$$
\begin{aligned}
Y_t &= A_t(k_{Tt}^{\theta} l_{Tt}^{1-\theta})^{\nu} X^{1-\nu} \\
P_t &= (1-\nu)\frac{Y_t}{X_t}
\end{aligned}
$$

Output on intermediate good sector in the home country

$$
Y_{It} = \left[y_{dt}^{\eta} + (y_{ft}^{*})^{\eta} \right]^{\frac{1}{\eta}}
$$

Optimality conditions w.r.t. labor for home firms

$$
\begin{aligned}
0 &= (1-\theta)\nu Y_t - w_t l_{Tt} \\
0 &= (1-\theta)(1-\phi)\eta P_t Y_{It}^{1-\eta} y_{dt}^{\eta} - w_t l_{dt} \\
0 &= (1-\theta)(1-\phi)\eta P_t^{*} (Y_{It}^{*})^{1-\eta} y_{ft}^{\eta} - w^{*} t l_{ft}
\end{aligned}
$$

Value of an additional unit of investment (i.e. Lagrange multiplier on investment accumulation equation) for home firms

$$
\begin{aligned}
V_{Tt} &= \left[\chi_1 \left(\frac{i_{Tt}}{k_{Tt-1}} \right)^{-\psi} \right]^{-1} \\
V_{dt} &= \left[\chi_1 \left(\frac{i_{dt}}{k_{dt-1}} \right)^{-\psi} \right]^{-1} \\
V_{ft} &= \left[\chi_1 \left(\frac{i_{ft}}{k_{ft-1}} \right)^{-\psi} \right]^{-1} \\
V_{Mt} &= \left[\chi_1^{m} \left(\frac{i_{Mt}}{M_{t-1}} \right)^{-\psi_m} \right]^{-1}
\end{aligned}
$$

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Optimality conditions with respect to investment, home firms

$$
\begin{aligned}
V_{Tt}k_{Tt+1} &= \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\theta\eta Y_{t+1} - i_{Tt+1} + V_{Tt+1}k_{Tt+2} \right) \right\} \\
V_{dt}k_{dt+1} &= \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\theta(1-\phi)\eta P_t Y_{It}^{1-\eta} y_{dt}^{\eta} - i_{dt+1} + V_{dt+1}k_{dt+2} \right) \right\} \\
V_{ft}k_{ft+1} &= \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\theta(1-\phi)\eta P_t^* (Y_{It}^*)^{1-\eta} y_{ft}^{\eta} - i_{ft+1} + V_{ft+1}k_{ft+2} \right) \right\} \\
V_{Mt}M_{t+1} &= \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[\phi\eta \left(P_t Y_{It}^{1-\eta} y_{dt}^{\eta} + P_t^* (Y_{It}^*)^{1-\eta} y_{ft}^{\eta} \right) - i_{Mt+1} + V_{Mt+1}M_{t+2} \right] \right\}
\end{aligned}
$$

Domestic and foreign production by home owned multinationals

$$
\begin{aligned}
y_{dt} &= e^{z_t} M_t^{\phi} \left(k_{dt}^{\theta} l_{dt}^{1-\theta} \right)^{1-\phi} \\
y_{ft} &= \tau e^{z_t} M_t^{\phi} \left(k_{ft}^{\theta} l_{ft}^{1-\theta} \right)^{1-\phi}
\end{aligned}
$$

Total dividends households in the home country receive

$$
\begin{aligned}
d_t = (Y_t - w_t l_{Tt} - i_{Tt}) &+ (P_t Y_{It}^{1-\eta} y_{dt}^{\eta} - w_t l_{dt} - i_{dt}) \\
&+ (P_t^* (Y_{It}^*)^{1-\eta} y_{ft}^{\eta} - w_t^* l_{ft} - i_{ft}) - i_{Mt}
\end{aligned}
$$

Capital accumulation

$$
\begin{aligned}
k_{jt+1} &= (1-\delta)k_{jt} + \left[\frac{\chi_1 \left(\frac{i_{jt}}{k_{jt}} \right)^{1-\psi}}{1-\psi} + \chi_2 \right] k_{jt} \qquad j = T, d, f \\
M_{t+1} &= (1-\delta_m)M_t + \left[\frac{\chi_1^m \left(\frac{i_{Mt}}{M_t} \right)^{1-\psi_m}}{1-\psi_m} + \chi_2^m \right] M_t
\end{aligned}
$$

Analoguous for foreign firms. Domestic and foreign households

$$
\begin{aligned}
\lambda_t &= U_1(C_t, \bar{L} - L_t) \\
\lambda_t^* &= U_1(C_t^*, \bar{L} - L_t^*) \\
0 &= U_2(C_t, \bar{L} - L_t) + \lambda_t w_t \\
0 &= U_2(C_t^*, \bar{L} - L_t^*) + \lambda_t^* w_t^*
\end{aligned}
$$

Labor market clearing

$$
\begin{aligned}
L_t &= l_{Tt} + l_{dt} + l_{ft}^* \\
L_t^* &= l_{Tt}^* + l_{dt}^* + l_{ft}
\end{aligned}
$$

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Goods market clearing

$$Y_t + Y_t^* = C_t + C_t^* + i_{Tt} + i_{Tt}^* + i_{dt} + i_{ft} + i_{dt}^* + i_{ft}^* + i_{Mt} + i_{Mt}^*$$

International financial markets:

a) Complete financial markets

$$\lambda_t = \lambda_t^*$$

b) Financial autarky

$$C_t = w_t L_t + d_{Tt} + d_{Mt}$$

B.3.2 Deterministic steady state

The system of equations in the deterministic steady state for two complete open economies $\tau = 1$ are given by

$$\frac{1-\beta(1-\delta)}{\nu\theta\beta} = \frac{Y}{K_1} \qquad \frac{1-\beta(1-\delta)}{\nu\theta\beta} = \frac{Y^*}{K_1^*}$$

$$\frac{1-\beta(1-\delta)}{(1-\phi)\theta\eta\beta} = PY_2^{1-\eta}\frac{y_d^\eta}{k_d} \qquad \frac{1-\beta(1-\delta)}{(1-\phi)\theta\eta\beta} = PY_2^{1-\eta}\frac{(y_f^*)^\eta}{k_f^*}$$

$$\frac{1-\beta(1-\delta)}{(1-\phi)\theta\eta\beta} = P^*(Y_2^*)^{1-\eta}\frac{y_f^\eta}{k_f} \qquad \frac{1-\beta(1-\delta)}{(1-\phi)\theta\eta\beta} = P^*(Y_2^*)^{1-\eta}\frac{(y_d^*)^\eta}{k_d^*}$$

$$\frac{1-\beta(1-\delta_m)}{\phi\eta\beta} = \frac{PY_2^{1-\eta}y_d^\eta + P^*(Y_2^*)^{1-\eta}y_f^\eta}{M} \qquad \frac{1-\beta(1-\delta_m)}{\phi\eta\beta} = \frac{P^*(Y_2^*)^{1-\eta}(y_d^*)^\eta + PY_2^{1-\eta}(y_f^*)^\eta}{M^*}$$

$$\begin{aligned}
wL_1 &= (1-\theta)\nu Y & wL_1^* &= (1-\theta)\nu Y^* \\
wl_d &= (1-\theta)(1-\phi)\eta PY_2^{1-\eta}y_d^\eta & wl_f^* &= (1-\theta)(1-\phi)\eta PY_2^{1-\eta}(y_f^*)^\eta \\
w^*l_d^* &= (1-\theta)(1-\phi)\eta P^*(Y_2^*)^{1-\eta}(y_d^*)^\eta & w^*l_f &= (1-\theta)(1-\phi)\eta P^*(Y_2^*)^{1-\eta}y_f^\eta \\
L &= l_f^* + l_d + L_1 & L^* &= L_1^* + l_d^* + l_f \\
K &= k_f^* + k_d + K_1 & K^* &= K_1^* + k_d^* + k_f \\
Y &= (K_1^\theta L_1^{1-\theta})^\nu Y_2^{1-\nu} & Y^* &= ((K_1^*)^\theta (L_1^*)^{1-\theta})^\nu (Y_2^*)^{1-\nu} \\
P &= (1-\nu)Y & P^* &= (1-\nu)Y^*
\end{aligned}$$

$$c + c^* + \delta(K + K^*) + \delta_m(M + M^*) = Y + Y^*$$

Note that when $\tau = 1$ both countries are identical, therefore all home and foreign quantities and prices are identical, so I will omit for now the asterisk. One can show that

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$(k_d + k_f)/K_1 = (l_d + l_f)/L_1 = \xi$, with $\xi = (1-\phi)(1-\nu)\eta/\nu$. Also note that the parameters of the utility function are set so that $L = 1$ in steady state, so that we obtain the following system of equation for technology, physical capital and aggregate output:

$$
\begin{aligned}
M &= \frac{\phi\beta(1-\nu)\eta}{1-\beta(1-\delta_m)} Y \\
K &= \frac{(1-\phi)\theta\beta(1-\nu)\eta}{1-\beta(1-\delta)} Y \\
Y &= 2^{1-\nu} \left(\frac{1}{1+\xi}\right)^{\nu} \left(\frac{\xi}{1+\xi}\right)^{(1-\phi)(1-\nu)} K^{\theta(\nu+(1-\nu)(1-\phi))}
\end{aligned}
$$

We can combine these equations and obtain the steady state values for technology and physical capital

$$
\begin{aligned}
M &= \zeta_1^{\frac{\theta(\nu+(1-\nu)(1-\phi)}{\zeta_3}} \zeta_2^{\frac{1}{\zeta_3}} \\
K &= \zeta_1 M
\end{aligned}
$$

with

$$
\begin{aligned}
\zeta_1 &\equiv \frac{\theta(\nu+(1-\nu)(1-\phi)\eta)}{\phi(1-\nu)\eta} \frac{1-\beta(1-\delta_m)}{1-\beta(1-\delta)} \\
\zeta_2 &\equiv \frac{\phi\beta(1-\nu)\eta}{1-\beta(1-\delta_m)} 2^{1-\nu} \left(\frac{1}{1+\xi}\right)^{\nu} \left(\frac{\xi}{1+\xi}\right)^{(1-\phi)(1-\nu)} \\
\zeta_3 &\equiv 1-\phi(1-\nu)-\theta(\nu+(1-\phi)(1-\nu)).
\end{aligned}
$$

All the other quantities follow by plugging in these values in the respective equations. Because financial market structure only in the economy with uncertainty, the deterministic steady state is the same whether or not financial markets are complete.

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